

## Properties of Determinants

Property-1: The value of the determinant remains unchanged if its rows and columns are interchanged.

Remark: It follows from above property that if  $A$  is a square matrix, then  $\det(A) = \det(A')$ , where  $A'$  = Transpose of  $A$ .

Property-2: If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

Property-3: If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then value of determinant is zero.

Property-4: If each element of a row (or a column) of a determinant is multiplied by a constant  $k$ , then its value gets multiplied by  $k$ .

Remarks:

i) By this property, we can take out any common factor from any one row or any one column of a given determinant.

ii) If corresponding elements of any two rows (or a column) of a determinant ~~is~~ multiplied are proportional (in the same ratio), then its value is zero.

Property-5: If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

Property-6: If, to each element of any row or column of a determinant, the equimultiples of corresponding elements of other row (or column) are added, then value of determinant remains the same, i.e. the value of determinant remain same if we apply the operation  $R_i \rightarrow R_i + kR_j$  or  $C_i \rightarrow C_i + kC_j$

### Remarks:

- i) If  $\Delta_1$  is the determinant obtained by applying  $R_i \rightarrow kR_i$  or  $C_i \rightarrow kC_i$  to the determinant of  $\Delta$ , then  $\Delta_1 = k\Delta$
- ii) If more than one operation like  $R_i \rightarrow R_i + kR_j$  is done in one step, care should be taken to see that a row that is affected in one operation should not be used in another operation. A similar remark applies to column operations.