spheres. The neutral point N (see Fig. 8.10) is defined as the position where the two forces cancel each other exactly. If ON = r, we have

$$\frac{GMm}{r^2} = \frac{4GMm}{(6R-r)^2}$$

$(6R - r)^2 = 4r^2$

$6R - r = \pm 2r$

$r = 2R$ or $-6R$.

The neutral point $r = -6R$ does not concern us in this example. Thus ON = r = $2R$. It is sufficient to project the particle with a speed which would enable it to reach N. Thereafter, the greater gravitational pull of $4M$ would suffice. The mechanical energy at the surface of M is

$$E_i = \frac{1}{2} m v^2 - \frac{GMm}{R} - \frac{4GMm}{5R}.$$

At the neutral point N, the speed approaches zero. The mechanical energy at N is purely potential.

$$E_N = -\frac{GMm}{2R} - \frac{4GMm}{4R}.$$

From the principle of conservation of mechanical energy

$$\frac{1}{2}v^2 - \frac{GM}{R} - \frac{4GM}{5R} = -\frac{GM}{2R} - \frac{GM}{R}$$

or

$$v^2 = \frac{2GM}{R}\left(\frac{4}{5} - \frac{1}{2}\right)$$

$$v = \left(\frac{3GM}{5R}\right)^{1/2}$$ ◀

A point to note is that the speed of the projectile is zero at N, but is nonzero when it strikes the heavier sphere 4 M. The calculation of this speed is left as an exercise to the students.

8.9 EARTH SATELLITES

Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them. In particular, their orbits around the earth are circular or elliptic. Moon is the only natural satellite of the earth with a near circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis. Since, 1957, advances in technology have enabled many countries including India to launch artificial earth satellites for practical use in fields like telecommunication, geophysics and meteorology.

We will consider a satellite in a circular orbit of a distance $(R_E + h)$ from the centre of the earth, where R_E = radius of the earth. If m is the mass of the satellite and V its speed, the centripetal force required for this orbit is

$$\text{F(centripetal)} = \frac{mV^2}{(R_E + h)} \qquad (8.33)$$

directed towards the center. This centripetal force is provided by the gravitational force, which is

$$\text{F(gravitation)} = \frac{G m M_E}{(R_E + h)^2} \qquad (8.34)$$

Where M_E is the mass of the earth.

Equating R.H.S of Eqs. (8.33) and (8.34) and cancelling out m, we get

$$V^2 = \frac{GM_E}{(R_E + h)} \qquad (8.35)$$

Thus V decreases as h increases. From equation (8.35),the speed V for $h = 0$ is

$$V^2 \ (h = 0) = GM/R_E = gR_E \qquad (8.36)$$

where we have used the relation $g = GM/R_E^2$. In every orbit, the satellite traverses a distance $2\pi(R_E + h)$ with speed V. It's time period T therefore is

$$T = \frac{2\pi(R_E + h)}{V} = \frac{2\pi(R_E + h)^{3/2}}{\sqrt{GM_E}} \qquad (8.37)$$

on substitution of value of V from Eq. (8.35). Squaring both sides of Eq. (8.37), we get

$$T^2 = k \ (R_E + h)^3 \ \ (\text{where } k = 4\pi^2 / GM_E) \qquad (8.38)$$

which is Kepler's law of periods, as applied to motion of satellites around the earth. For a satellite very close to the surface of earth h can be neglected in comparison to R_E in Eq. (8.38). Hence, for such satellites, T is T_o, where

$$T_0 = 2\pi\sqrt{R_E / g} \qquad (8.39)$$

If we substitute the numerical values $g \simeq 9.8$ m s^{-2} and R_E = 6400 km., we get

$$T_0 = 2\pi\sqrt{\frac{6.4 \times 10^6}{9.8}} \text{ s}$$

Which is approximately 85 minutes.

▶ **Example 8.5** The planet Mars has two moons, phobos and delmos. (i) phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4×10^3 km. Calculate the mass of mars. (ii) Assume that earth and mars move in circular orbits around the sun, with the martian orbit being 1.52 times the orbital radius of the earth. What is the length of the martian year in days ?

Answer (i) We employ Eq. (8.38) with the sun's mass replaced by the martian mass M_m

$$T^2 = \frac{4\pi^2}{GM_m} R^3$$

$$M_m = \frac{4\pi^2}{G} \frac{R^3}{T^2}$$

$$= \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times 10^{-11} \times (459 \times 60)^2}$$

$$M_m = \frac{4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}}{6.67 \times (4.59 \times 6)^2 \times 10^{-5}}$$

$= 6.48 \times 10^{23}$ kg.

(ii) Once again Kepler's third law comes to our aid,

$$\frac{T_M^2}{T_E^2} = \frac{R_{MS}^3}{R_{ES}^3}$$

where R_{MS} is the mars -sun distance and R_{ES} is the earth-sun distance.

$\therefore T_M = (1.52)^{3/2} \times 365$

$= 684$ days

We note that the orbits of all planets except Mercury, Mars and Pluto are very close to being circular. For example, the ratio of the semi-minor to semi-major axis for our Earth is, $b/a = 0.99986$. ◀

▶ **Example 8.6 Weighing the Earth** : You are given the following data: $g = 9.81$ ms^{-2}, $R_E = 6.37 \times 10^6$ m, the distance to the moon $R = 3.84 \times 10^8$ m and the time period of the moon's revolution is 27.3 days. Obtain the mass of the Earth M_E in two different ways.

Answer From Eq. (8.12) we have

$$M_E = \frac{g R_E^2}{G}$$

$$= \frac{9.81 \times \left(6.37 \times 10^6\right)^2}{6.67 \times 10^{-11}}$$

$= 5.97 \times 10^{24}$ kg.

The moon is a satellite of the Earth. From the derivation of Kepler's third law [see Eq. (8.38)]

$$T^2 = \frac{4\pi^2 R^3}{G M_E}$$

$$M_E = \frac{4\pi^2 R^3}{G T^2}$$

$$= \frac{4 \times 3.14 \times 3.14 \times (3.84)^3 \times 10^{24}}{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2}$$

$= 6.02 \times 10^{24}$ kg

Both methods yield almost the same answer, the difference between them being less than 1%. ◀

▶ **Example 8.7** Express the constant k of Eq. (8.38) in days and kilometres. Given $k = 10^{-13}$ s^2 m^{-3}. The moon is at a distance of 3.84×10^5 km from the earth. Obtain its time-period of revolution in days.

Answer Given

$k = 10^{-13}$ s^2 m^{-3}

$$= 10^{-13}\left[\frac{1}{(24 \times 60 \times 60)^2} \mathrm{d}^2\right]\left[\frac{1}{(1/1000)^3 \mathrm{km}^3}\right]$$

$= 1.33 \times 10^{-14}$ d^2 km^{-3}

Using Eq. (8.38) and the given value of k, the time period of the moon is

$T^2 = (1.33 \times 10^{-14})(3.84 \times 10^5)^3$

$T = 27.3$ d ◀

Note that Eq. (8.38) also holds for elliptical orbits if we replace (R_E+h) by the semi-major axis of the ellipse. The earth will then be at one of the foci of this ellipse.

8.10 ENERGY OF AN ORBITING SATELLITE

Using Eq. (8.35), the kinetic energy of the satellite in a circular orbit with speed v is

$$KgE = \frac{1}{2} m v^2$$

$$= \frac{Gm M_E}{2(R_E + h)}, \quad (8.40)$$

Considering gravitational potential energy at infinity to be zero, the potential energy at distance (R_e+h) from the center of the earth is

$$P.E = -\frac{G m M_E}{(R_E + h)} \quad (8.41)$$

The K.E is positive whereas the P.E is negative. However, in magnitude the K.E. is half the P.E, so that the total E is

$$E = K.E + P.E = -\frac{G m M_E}{2(R_E + h)} \quad (8.42)$$

The total energy of an circularly orbiting satellite is thus negative, with the potential energy being negative but twice is magnitude of the positive kinetic energy.

When the orbit of a satellite becomes elliptic, both the *K.E.* and *P.E.* vary from point to point. The total energy which remains constant is negative as in the circular orbit case. This is what we expect, since as we have discussed before if the total energy is positive or zero, the object escapes to infinity. Satellites are always at finite distance from the earth and hence their energies cannot be positive or zero.

▶ ***Example 8.8*** A 400 kg satellite is in a circular orbit of radius $2R_E$ about the Earth. How much energy is required to transfer it to a circular orbit of radius $4R_E$? What are the changes in the kinetic and potential energies ?

Answer Initially,

$$E_i = -\frac{G M_E m}{4 R_E}$$

While finally

$$E_f = -\frac{G M_E m}{8 R_E}$$

The change in the total energy is

$$\Delta E = E_f - E_i$$

$$= \frac{G M_E m}{8 R_E} = \left(\frac{G M_E}{R_E^2}\right)\frac{m R_E}{8}$$

$$\Delta E = \frac{g m R_E}{8} = \frac{9.81 \times 400 \times 6.37 \times 10^6}{8} = 3.13 \times 10^9 \text{ J}$$

The kinetic energy is reduced and it mimics ΔE, namely, $\Delta K = K_f - K_i = -3.13 \times 10^9$ J.

The change in potential energy is twice the change in the total energy, namely

$\Delta V = V_f - V_i = -6.25 \times 10^9$ J ◀

8.11 GEOSTATIONARY AND POLAR SATELLITES

An interesting phenomenon arises if in we arrange the value of (R_E+ h) such that T in Eq. (8.37) becomes equal to 24 hours. If the circular orbit is in the equatorial plane of the earth, such a satellite, having the same period as the period of rotation of the earth about its own axis would appear stationery viewed from a point on earth. The (R_E + h) for this purpose works out to be large as compared to R_E :

$$R_E + h = \left(\frac{T^2 G M_E}{4\pi^2}\right)^{1/3} \quad (8.43)$$

and for T = 24 hours, h works out to be 35800 km. which is much larger than R_E. Satellites in a circular orbits around the earth in the

India's Leap into Space

India entered the space age with the launching of the low orbit satellite Aryabhatta in 1975. In the first few years of its programme the launch vehicles were provided by the erstwhile Soviet Union. Indigenous launch vehicles were employed in the early 1980's to send the Rohini series of satellites into space. The programme to send polar satellites into space began in late 1980's. A series of satellites labelled IRS (Indian Remote Sensing Satellites) have been launched and this programme is expected to continue in future. The satellites have been employed for surveying, weather prediction and for carrying out experiments in space. The INSAT (Indian National Satellite) series of satellites were designed and made operational for communications and weather prediction purposes beginning in 1982. European launch vehicles have been employed in the INSAT series. India tested its geostationary launch capability in 2001 when it sent an experimental communications satellite (GSAT-1) into space. In 1984 Rakesh Sharma became the first Indian astronaut. The Indian Space Research Organisation (ISRO) is the umbrella organisation that runs a number of centre. Its main lauch centre at Sriharikota (SHAR) is 100 km north of Chennai. The National Remote Sensing Agency (NRSA) is near Hyderabad. Its national centre for research in space and allied sciences is the Physical Research Laboratory (PRL) at Ahmedabad.

equatorial plane with T = 24 hours are called Geostationery Satellites. Clearly, since the earth rotates with the same period, the satellite would appear fixed from any point on earth. It takes very powerful rockets to throw up a satellite to such large heights above the earth but this has been done in view of the several benefits of many practical applications.

It is known that electromagnetic waves above a certain frequency are not reflected from ionosphere. Radio waves used for radio broadcast which are in the frequency range 2 MHz to 10 MHz, are below the critical frequency. They are therefore reflected by the ionosphere. Thus radio waves broadcast from an antenna can be received at points far away where the direct wave fail to reach on account of the curvature of the earth. Waves used in television broadcast or other forms of communication have much higher frequencies and thus cannot be received beyond the line of sight. A Geostationery satellite, appearing fixed above the broadcasting station can however receive these signals and broadcast them back to a wide area on earth. The INSAT group of satellites sent up by India are one such group of Geostationary satellites widely used for telecommunications in India.

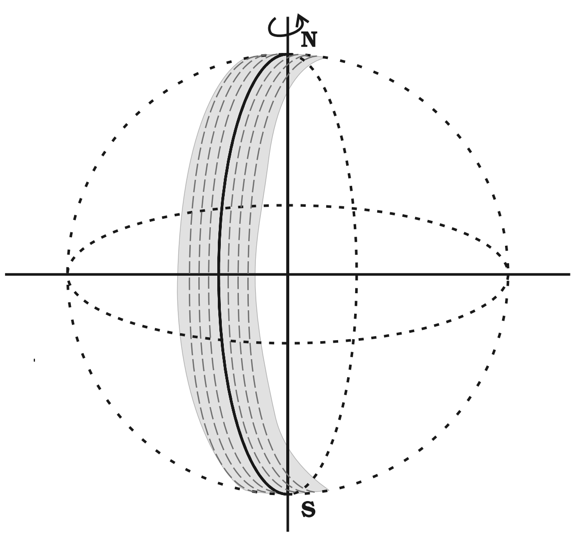

Fig. 8.11 *A Polar satellite. A strip on earth's surface (shown shaded) is visible from the satellite during one cycle. For the next revolution of the satellite, the earth has rotated a little on its axis so that an adjacent strip becomes visible.*

Another class of satellites are called the Polar satellites (Fig. 8.11). These are low altitude ($h \approx$ 500 to 800 km) satellites, but they go around the poles of the earth in a north-south direction whereas the earth rotates around its axis in an east-west direction. Since its time period is around 100 minutes it crosses any altitude many times a day. However, since its height h above the earth is about 500-800 km, a camera fixed on it can view only small strips of the earth in one orbit. Adjacent strips are viewed in the next orbit, so that in effect the whole earth can be viewed strip by strip during the entire day. These satellites can view polar and equatorial regions at close distances with good resolution. Information gathered from such satellites is extremely useful for remote sensing, meterology as well as for environmental studies of the earth.

8.12 WEIGHTLESSNESS

Weight of an object is the force with which the earth attracts it. We are conscious of our own weight when we stand on a surface, since the surface exerts a force opposite to our weight to keep us at rest. The same principle holds good when we measure the weight of an object by a spring balance hung from a fixed point e.g. the ceiling. The object would fall down unless it is subject to a force opposite to gravity. This is exactly what the spring exerts on the object. This is because the spring is pulled down a little by the gravitational pull of the object and in turn the spring exerts a force on the object vertically upwards.

Now, imagine that the top end of the balance is no longer held fixed to the top ceiling of the room. Both ends of the spring as well as the object move with identical acceleration g. The spring is not stretched and does not exert any upward force on the object which is moving down with acceleration g due to gravity. The reading recorded in the spring balance is zero since the spring is not stretched at all. If the object were a human being, he or she will not feel his weight since there is no upward force on him. Thus, when an object is in free fall, it is weightless and this phenomenon is usually called the phenomenon of weightlessness.

In a satellite around the earth, every part and parcel of the satellite has an acceleration towards the center of the earth which is exactly the value of earth's acceleration due to gravity at that position. Thus in the satellite everything inside it is in a state of free fall. This is just as if we were falling towards the earth from a height. Thus, in a manned satellite, people inside experience no gravity. Gravity for us defines the vertical direction and thus for them there are no horizontal or vertical directions, all directions are the same. Pictures of astronauts floating in a satellite reflect show this fact.

SUMMARY

1. Newton's law of universal gravitation states that the gravitational force of attraction between any two particles of masses m_1 and m_2 separated by a distance r has the magnitude

$$F = G\frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant, which has the value 6.672×10^{-11} N m^2 kg^{-2}.

2. If we have to find the resultant gravitational force acting on the particle m due to a number of masses $M_1, M_2, \ldots M_n$ etc. we use the principle of superposition. Let $F_1, F_2, \ldots F_n$ be the individual forces due to $M_1, M_2, \ldots M_n$, each given by the law of gravitation. From the principle of superposition each force acts independently and uninfluenced by the other bodies. The resultant force F_R is then found by vector addition

$$F_R = F_1 + F_2 + \ldots + F_n = \sum_{i=1}^{n} \boldsymbol{F_i}$$

where the symbol 'Σ' stands for summation.

3. Kepler's laws of planetary motion state that
   (a) All planets move in elliptical orbits with the Sun at one of the focal points
   (b) The radius vector drawn from the sun to a planet sweeps out equal areas in equal time intervals. This follows from the fact that the force of gravitation on the planet is central and hence angular momentum is conserved.
   (c) The square of the orbital period of a planet is proportional to the cube of the semi-major axis of the elliptical orbit of the planet

   The period T and radius R of the circular orbit of a planet about the Sun are related by

$$T^2 = \left(\frac{4\pi^2}{G\,M_s}\right)R^3$$

   where M_s is the mass of the Sun. Most planets have nearly circular orbits about the Sun. For elliptical orbits, the above equation is valid if R is replaced by the semi-major axis, a.

4. The acceleration due to gravity.
   (a) at a height h above the Earth's surface

$$g(h) = \frac{G\,M_E}{(R_E + h)^2}$$

$$\approx \frac{G\,M_E}{R_E^2}\left(1 - \frac{2h}{R_E}\right) \quad \text{for } h \ll R_E$$

$$g(h) = g(0)\left(1 - \frac{2h}{R_E}\right) \quad \text{where } g(0) = \frac{G\,M_E}{R_E^2}$$