Copernicus (1473-1543) proposed a definitive model in which the planets moved in circles around a fixed central sun. His theory was discredited by the church, but notable amongst its supporters was Galileo who had to face prosecution from the state for his beliefs.

It was around the same time as Galileo, a nobleman called Tycho Brahe (1546-1601) hailing from Denmark, spent his entire lifetime recording observations of the planets with the naked eye. His compiled data were analysed later by his assistant Johannes Kepler (1571-1640). He could extract from the data three elegant laws that now go by the name of Kepler's laws. These laws were known to Newton and enabled him to make a great scientific leap in proposing his universal law of gravitation.

8.2 KEPLER'S LAWS

The three laws of Kepler can be stated as follows:

1. Law of orbits : All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse (Fig. 8.1a). This law was a deviation from the Copernican model which allowed only circular orbits. The ellipse, of which the circle is a special case, is a closed curve which can be drawn very simply as follows.

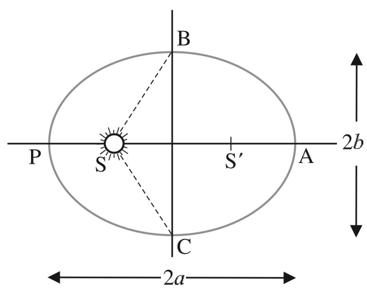

Fig. 8.1(a) An ellipse traced out by a planet around the sun. The closest point is P and the farthest point is A, P is called the perihelion and A the aphelion. The semimajor axis is half the distance AP.

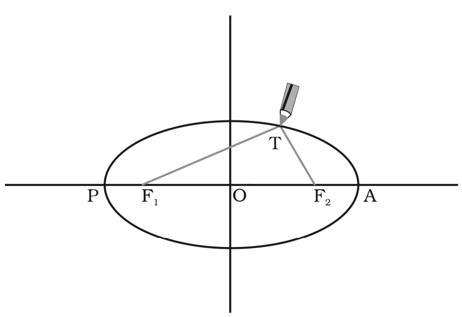

Fig. 8.1(b) Drawing an ellipse. A string has its ends fixed at F_1 and F_2. The tip of a pencil holds the string taut and is moved around.

Select two points F_1 and F_2. Take a length of a string and fix its ends at F_1 and F_2 by pins. With the tip of a pencil stretch the string taut and then draw a curve by moving the pencil keeping the string taut throughout.(Fig. 8.1(b)) The closed curve you get is called an ellipse. Clearly for any point T on the ellipse, the sum of the distances from F_1 and F_2 is a constant. F_1, F_2 are called the focii. Join the points F_1 and F_2 and extend the line to intersect the ellipse at points P and A as shown in Fig. 8.1(b). The midpoint of the line PA is the centre of the ellipse O and the length PO = AO is called the semi-major axis of the ellipse. For a circle, the two focii merge into one and the semi-major axis becomes the radius of the circle.

2. Law of areas : The line that joins any planet to the sun sweeps equal areas in equal intervals of time (Fig. 8.2). This law comes from the observations that planets appear to move slower when they are farther from the sun than when they are nearer.

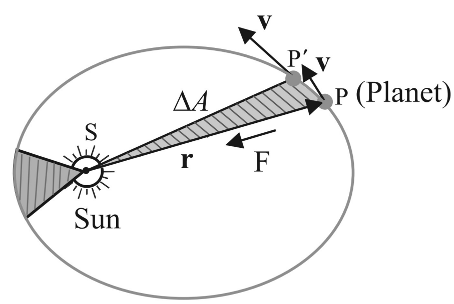

Fig. 8.2 The planet P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt.

3. Law of periods : The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.

The table below gives the approximate time periods of revolution of nine planets around the sun along with values of their semi-major axes.

Table 1 **Data from measurement of planetary motions given below confirm Kepler's Law of Periods**

a ≡ Semi-major axis in units of 10^{10} m.
T ≡ Time period of revolution of the planet in years(y).
Q ≡ The quotient (T^2/a^3) in units of 10^{-34} y^2 m^{-3}.

Planet	a	T	Q
Mercury	5.79	0.24	2.95
Venus	10.8	0.615	3.00
Earth	15.0	1	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

The law of areas can be understood as a consequence of conservation of angular momentum whch is valid for any central force . A central force is such that the force on the planet is along the vector joining the sun and the planet. Let the sun be at the origin and let the position and momentum of the planet be denoted by **r** and **p** respectively. Then the area swept out by the planet of mass m in time interval Δt is (Fig. 8.2) $\Delta\mathbf{A}$ given by

$$\Delta\mathbf{A} = \tfrac{1}{2}\ (\mathbf{r} \times \mathbf{v}\Delta t) \tag{8.1}$$

Hence

$$\Delta\mathbf{A}/\Delta t = \tfrac{1}{2}\ (\mathbf{r} \times \mathbf{p})/\mathrm{m},\ (\text{since } \mathbf{v} = \mathbf{p}/\mathrm{m})$$
$$= \mathbf{L} / (2\ \mathrm{m}) \tag{8.2}$$

where **v** is the velocity, **L** is the angular momentum equal to (**r** × **p**). For a central force, which is directed along **r**, **L** is a constant

Johannes Kepler (1571–1630) was a scientist of German origin. He formulated the three laws of planetary motion based on the painstaking observations of Tycho Brahe and coworkers. Kepler himself was an assistant to Brahe and it took him sixteen long years to arrive at the three planetary laws. He is also known as the founder of geometrical optics, being the first to describe what happens to light after it enters a telescope.

as the planet goes around. Hence, $\Delta\mathbf{A}/\Delta t$ is a constant according to the last equation. This is the law of areas. Gravitation is a central force and hence the law of areas follows.

▶ ***Example 8.1*** Let the speed of the planet at the perihelion *P* in Fig. 8.1(a) be v_P and the Sun-planet distance SP be r_P. Relate $\{r_P, v_P\}$ to the corresponding quantities at the aphelion $\{r_A, v_A\}$. Will the planet take equal times to traverse *BAC* and *CPB* ?

Answer The magnitude of the angular momentum at *P* is $L_p = m_p r_p v_p$, since inspection tells us that $\mathbf{r}_p$ and $\mathbf{v}_p$ are mutually perpendicular. Similarly, $L_A = m_p r_A v_A$. From angular momentum conservation

$$m_p r_p v_p = m_p r_A v_A$$

or

$$\frac{v_p}{v_A} = \frac{r_A}{r_p}$$ ◀

Since $r_A > r_p$, $v_p > v_A$.

The area *SBAC* bounded by the ellipse and the radius vectors *SB* and *SC* is larger than SBPC in Fig. 8.1. From Kepler's second law, equal areas are swept in equal times. Hence the planet will take a longer time to traverse *BAC* than *CPB*.

8.3 UNIVERSAL LAW OF GRAVITATION

Legend has it that observing an apple falling from a tree, Newton was inspired to arrive at an universal law of gravitation that led to an explanation of terrestrial gravitation as well as of Kepler's laws. Newton's reasoning was that the moon revolving in an orbit of radius R_m was subject to a centripetal acceleration due to earth's gravity of magnitude

$$a_m = \frac{V^2}{R_m} = \frac{4\pi^2 R_m}{T^2} \tag{8.3}$$

where *V* is the speed of the moon related to the time period *T* by the relation $V = 2\pi R_m / T$. The time period *T* is about 27.3 days and R_m was already known then to be about 3.84×10^8m. If we substitute these numbers in equation (8.3), we get a value of a_m much smaller than the value of acceleration due to gravity g on the surface of the earth, arising also due to earth's gravitational attraction.