

**1. Employing Thomson's model, calculate the radius of a hydrogen atom and the wavelength of emitted light if the ionization energy of the atom is known to be equal to  $E = 13.6 \text{ eV}$ .**

Solution:

The Thomson model consists of a uniformly charged nucleus in which the electrons are at rest at certain equilibrium points (the plum in the pudding model). For the hydrogen nucleus, the charge on the nucleus is  $+e$  while the charge on the electron is  $-e$ . The electron by symmetry must be at the centre of the nuclear charge where the potential is

$$\phi_0 = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{3e}{2R}\right)$$

where  $R$  is the radius of the nuclear charge distribution. The potential energy of the electron is  $-e\phi_0$  and since the electron is at rest, and this is also the total energy. To ionise such an energetic electron will require an energy of  $E=e\phi_0$

From this we find

$$R = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{3e^2}{2E}\right)$$

In Gaussian system the factor

$\frac{1}{4\pi\epsilon_0}$   
is missing

Light is emitted when the electron vibrates. If we displace the electron slightly inside the nucleus by giving it a push  $r$  in some radial direction and an energy  $\delta E$  of oscillation then since the potential at a distance  $r$  in the nucleus is

$$\varphi(r) = \left( \frac{1}{4\pi\epsilon_0} \right) \frac{e}{R} \left( \frac{3}{2} - \frac{r^2}{2R^2} \right)$$

The total energy of the nucleus becomes

$$\frac{1}{2} m r^2 - \left( \frac{1}{4\pi\epsilon_0} \right) \frac{e^2}{R} \left( \frac{3}{2} - \frac{r^2}{2R^2} \right) = -e\varphi_0 + \delta E$$

$$\delta E = \frac{1}{2} m \dot{r}^2 + \left( \frac{1}{4\pi\epsilon_0} \right) \frac{e^2}{2R^3} r^2$$

This is the energy of a harmonic oscillator whose frequency is

$$\omega = \frac{1}{4\pi\epsilon_0} \frac{e^2}{mR^3}$$

The vibrating electron emits radiation of frequency  $\omega$  whose wavelength is

$$\lambda = \frac{2\pi c}{\omega} = \frac{2\pi c}{e} \sqrt{mR^3} (4\pi\epsilon_0)^{\frac{1}{2}}$$

In Gaussian units the factor

$$(4\pi\epsilon_0)^{\frac{1}{2}}$$

is missing

Putting the values we get  $\lambda = 0.237 \mu\text{m}$ .