4. An alpha particle with kinetic energy T = 0.50 MeV is deflected through an angle of θ = 90° by the Coulomb field of a stationary Hg nucleus. Find: All the formulas in this Part are given in the Gaussian system of units.

(a) the least curvature radius of its trajectory;

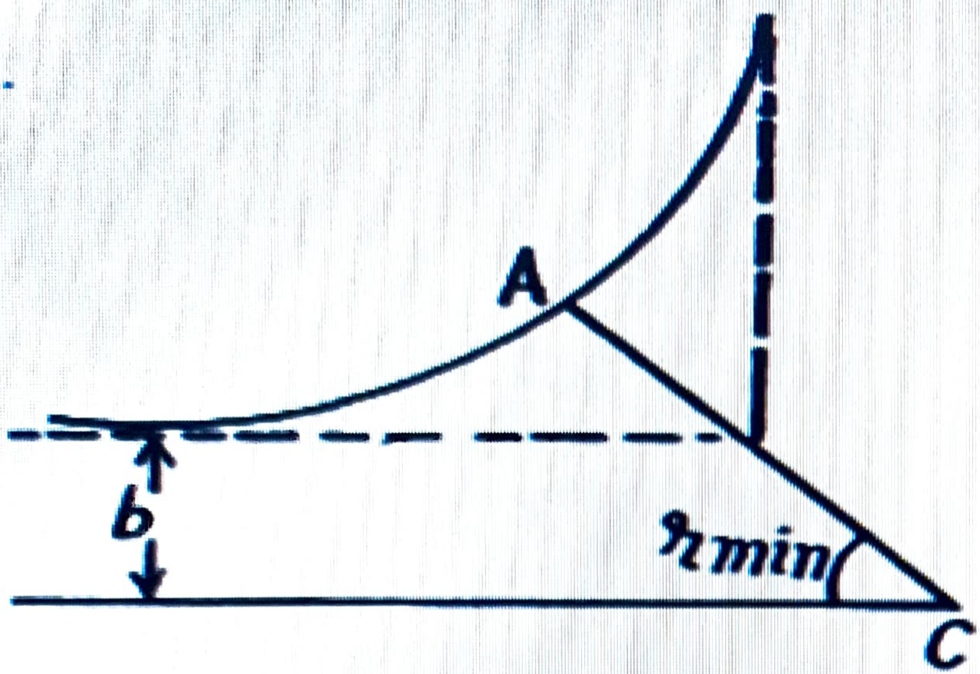
(b) the minimum approach distance between the particle and the nucleus

Solution:

We shall ignore the recoil of Hg nucleus.

(a) Let A be the point of closest approach to the centre C,AC=r_{min}

At A the motion is instantaneously circular because the radial velocity vanishes. Then if v_o the particle at A, the following equations hold equations hold



$$T = \frac{1}{2} m v_0^2 + \frac{Z_1 \; Z_2 \; e^2}{(4 \pi \varepsilon_0) r_{min}} \quad (1)$$

$$m v_0 r_{min} = \sqrt{2 m T} b \quad (2)$$

$$\frac{m v_0^2}{\rho_{min}} = \frac{Z_1 \; Z_2 \; e^2}{(4 \pi \varepsilon_0) r_{min}^2} \quad (3)$$

$$b = \frac{Z_1 \; Z_2 \; e^2}{(4 \pi \varepsilon_0) 2 T} \cot \frac{\theta}{2} \quad (4)$$

From 2 and 3

$$\frac{2 T b^2}{\rho_{min}} = \frac{Z_1 \; Z_2 \; e^2}{(4 \pi \varepsilon_0)}$$

$$\text{or, } \rho_{min} = \frac{Z_1 \; Z_2 \; e^2}{(4 \pi \varepsilon_0) 2 T} \cot^2 \frac{\theta}{2}$$

ρ_{min}= 0.231pm

*we used, $\delta=\delta_{min}$ is the radius of curvature of the path at A and P is minimum at A by symmetry and finally found equa

(4)

(b) From equations 2 and 4 we get

$$r_{min} = \frac{Z_1 Z_2 e^2}{(4\pi\varepsilon_0)\sqrt{2mT}} \frac{\cot\theta/2}{v_0},$$

Substituting in (1)

$$T = \frac{1}{2} m v_0^2 + \sqrt{2mT}\, v_0 \tan\theta/2$$

Solving for v_o we get

$$v_0 = \sqrt{\frac{2T}{m}}\left(\sec\frac{\theta}{2} - \tan\frac{\theta}{2}\right)$$

Then

$$r_{min} = \frac{Z_1 Z_2 e^2}{(4\pi\varepsilon_0) 2T} \frac{\cot\frac{\theta}{2}}{\sec\frac{\theta}{2} - \tan\frac{\theta}{2}}$$

$$= \frac{Z_1 Z_2 e^2}{(4\pi\varepsilon_0) 2T} \cot\frac{\theta}{2}\left(\sec\frac{\theta}{2} + \tan\frac{\theta}{2}\right)$$

$$= \frac{Z_1 Z_2 e^2}{(4\pi\varepsilon_0) 2T}\left(1 + \operatorname{cosec}\frac{\theta}{2}\right) = 0.557 \text{ pm}.$$