Suppose the potential energy between an electron and a proton at a distance r is given by $-ke^2/3r^3$. Use Bohr's theory to obtain energy level of such a hypothetical atom.

According to given situation for an electron revolving in an n^{th} orbit, the potential energy is given as

$$U = -\frac{ke^2}{3r_n^3} \qquad \text{(i)}$$

The centripetal force on the electron due to this is given as

$$F = -\frac{dU}{dr} = \frac{ke^2}{r_n^4} \qquad \text{(ii)}$$

If in n^{th} orbit electron revolves at speed v_n, then we have

$$\frac{mv_H^2}{r_n} = \frac{ke^2}{r_n^4} \quad \text{or} \quad mv_n^2 = \frac{ke^2}{r_n^3} \qquad \text{(iii)}$$

From Bohr's second postulate, we have

$$mv_n r_n = \frac{nh}{2\pi} \qquad \text{(iv)}$$

From Eqs. (iii) and (iv), we have

$$v_n = \frac{nh}{2\pi mr_n} \quad \text{and} \quad m\left(\frac{nh}{2\pi mr_n}\right)^2 = \frac{ke^2}{r_n^3}$$

$$\text{or} \quad r_n = \frac{4\pi^2 ke^2 m}{n^2 h^2} \quad \text{and} \quad v_n = \frac{n^3 h^3}{8\pi^3 km^2 e^2}$$

Now energy in n^{th} orbit is equal to negative of KE,

$$E_n = \frac{-1}{2} mv_n^2 = -\frac{ke^2}{2r_n^3}$$

$$= -\frac{1}{2} ke^2 \left(\frac{n^2 h^2}{4\pi^2 ke^2 m}\right)^3$$

$$= \frac{-n^2 h^2}{128 k^2 e^4 m^3}$$