We point out a common misconception created by newspapers and magazines. They mention food values in calories and urge us to restrict diet intake to below 2400 calories. What they should be saying is kilocalories (kcal) and not calories. A person consuming 2400 calories a day will soon starve to death! 1 food calorie is 1 kcal. ◀

### 6.10.6 The Principle of Conservation of Energy

We have seen that the total mechanical energy of the system is conserved if the forces doing work on it are conservative. If some of the forces involved are non-conservative, part of the mechanical energy may get transformed into other forms such as heat, light and sound. However, the total energy of an isolated system does not change, as long as one accounts for all forms of energy. Energy may be transformed from one form to another but the total energy of an isolated system remains constant. Energy can neither be created, nor destroyed.

Since the universe as a whole may be viewed as an isolated system, the total energy of the universe is constant. If one part of the universe loses energy, another part must gain an equal amount of energy.

The principle of conservation of energy cannot be proved. However, no violation of this principle has been observed. The concept of conservation and transformation of energy into various forms links together various branches of physics, chemistry and life sciences. It provides a unifying, enduring element in our scientific pursuits. From engineering point of view all electronic, communication and mechanical devices rely on some forms of energy transformation.

## 6.11 POWER

Often it is interesting to know not only the work done on an object, but also the rate at which this work is done. We say a person is physically fit if he not only climbs four floors of a building but climbs them fast. **Power** is defined as the time rate at which work is done or energy is transferred.

The average power of a force is defined as the ratio of the work, $W$, to the total time $t$ taken

$$P_{av} = \frac{W}{t}$$

The instantaneous power is defined as the limiting value of the average power as time interval approaches zero,

$$P = \frac{dW}{dt} \tag{6.21}$$

The work $dW$ done by a force F for a displacement $d\mathbf{r}$ is $dW = \mathbf{F}.d\mathbf{r}$. The instantaneous power can also be expressed as

$$P = \mathbf{F}.\frac{d\mathbf{r}}{dt}$$

$$= \mathbf{F}.\mathbf{v} \tag{6.22}$$

where **v** is the instantaneous velocity when the force is **F**.

Power, like work and energy, is a scalar quantity. Its dimensions are $[ML^2T^{-3}]$. In the SI, its unit is called a watt (W). The watt is 1 J $s^{-1}$. The unit of power is named after James Watt, one of the innovators of the steam engine in the eighteenth century.

There is another unit of power, namely the horse-power (hp)

1 hp = 746 W

This unit is still used to describe the output of automobiles, motorbikes, etc.

We encounter the unit watt when we buy electrical goods such as bulbs, heaters and refrigerators. A 100 watt bulb which is on for 10 hours uses 1 kilowatt hour (kWh) of energy.

100 (watt) $\times$ 10 (hour)
= 1000 watt hour
=1 kilowatt hour (kWh)
= $10^3$ (W) $\times$ 3600 (s)
= $3.6 \times 10^6$ J

Our electricity bills carry the energy consumption in units of kWh. Note that kWh is a unit of energy and not of power.

▶ ***Example 6.11*** An elevator can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2 m $s^{-1}$. The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power.

***Answer*** The downward force on the elevator is

$$F = m\,g + F_f = (1800 \times 10) + 4000 = 22000 \text{ N}$$

The motor must supply enough power to balance this force. Hence,

$$P = \mathbf{F}.\ \mathbf{v} = 22000 \times 2 = 44000 \text{ W} = 59 \text{ hp}$$ ◀

## 6.12 COLLISIONS

In physics we study motion (change in position). At the same time, we try to discover physical quantities, which do not change in a physical process. The laws of momentum and energy conservation are typical examples. In this section we shall apply these laws to a commonly encountered phenomena, namely collisions. Several games such as billiards, marbles or carrom involve collisions.We shall study the collision of two masses in an idealised form.

Consider two masses $m_1$ and $m_2$. The particle $m_1$ is moving with speed $v_{1i}$, the subscript '*i*' implying initial. We can cosider $m_2$ to be at rest. No loss of generality is involved in making such a selection. In this situation the mass $m_1$ collides with the stationary mass $m_2$ and this is depicted in Fig. 6.10.

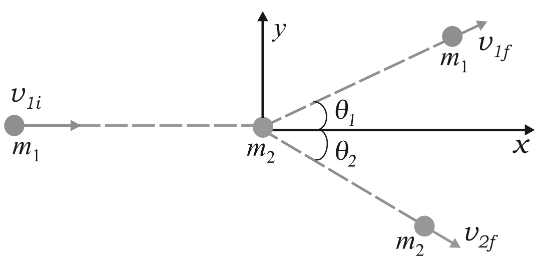



***Fig. 6.10*** *Collision of mass $m_1$, with a stationary mass $m_2$.*

The masses $m_1$ and $m_2$ fly-off in different directions. We shall see that there are relationships, which connect the masses, the velocities and the angles.

### 6.12.1 Elastic and Inelastic Collisions

In all collisions the total linear momentum is conserved; the initial momentum of the system is equal to the final momentum of the system. One can argue this as follows. When two objects collide, the mutual impulsive forces acting over the collision time $\Delta t$ cause a change in their respective momenta :

$$\Delta\mathbf{p}_1 = \mathbf{F}_{12}\ \Delta t$$
$$\Delta\mathbf{p}_2 = \mathbf{F}_{21}\ \Delta t$$

where $\mathbf{F}_{12}$ is the force exerted on the first particle by the second particle. $\mathbf{F}_{21}$ is likewise the force exerted on the second particle by the first particle. Now from Newton's Third Law, $\mathbf{F}_{12} = -\mathbf{F}_{21}$. This implies

$$\Delta\mathbf{p}_1 + \Delta\mathbf{p}_2 = \mathbf{0}$$

The above conclusion is true even though the forces vary in a complex fashion during the collision time $\Delta t$. Since the third law is true at every instant, the total impulse on the first object is equal and opposite to that on the second.

On the other hand, the total kinetic energy of the system is not necessarily conserved. The impact and deformation during collision may generate heat and sound. Part of the initial kinetic energy is transformed into other forms of energy. A useful way to visualise the deformation during collision is in terms of a 'compressed spring'. If the 'spring' connecting the two masses regains its original shape without loss in energy, then the initial kinetic energy is equal to the final kinetic energy but the kinetic energy during the collision time $\Delta t$ is not constant. Such a collision is called an **elastic collision**. On the other hand the deformation may not be relieved and the two bodies could move together after the collision. A collision in which the two particles move together after the collision is called a **completely inelastic collision**. The intermediate case where the deformation is partly relieved and some of the initial kinetic energy is lost is more common and is appropriately called an **inelastic collision**.

### 6.12.2 Collisions in One Dimension

Consider first a **completely inelastic collision** in one dimension. Then, in Fig. 6.10,

$$\theta_1 = \theta_2 = 0$$

$$m_1 v_{1i} = (m_1 + m_2) v_f \quad \text{(momentum conservation)}$$

$$v_f = \frac{m_1}{m_1 + m_2} v_{1i} \qquad (6.23)$$

The loss in kinetic energy on collision is

$$\Delta K = \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2) v_f^2$$

$$= \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} \frac{m_1^2}{m_1 + m_2} v_{1i}^2 \quad \text{[using Eq. (6.23)]}$$

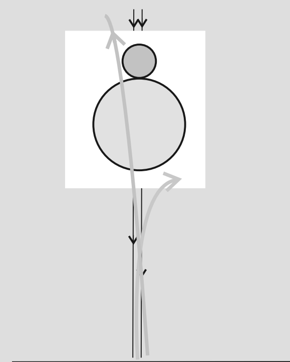
$$= \frac{1}{2} m_1 v_{1i}^2 \left[1 - \frac{m_1}{m_1 + m_2}\right]$$

**An experiment on head-on collision**

In performing an experiment on collision on a horizontal surface, we face three difficulties. One, there will be friction and bodies will not travel with uniform velocities. Two, if two bodies of different sizes collide on a table, it would be difficult to arrange them for a head-on collision unless their centres of mass are at the same height above the surface. Three, it will be fairly difficult to measure velocities of the two bodies just before and just after collision.

By performing this experiment in a vertical direction, all the three difficulties vanish. Take two balls, one of which is heavier (basketball/football/volleyball) and the other lighter (tennis ball/rubber ball/table tennis ball). First take only the heavier ball and drop it vertically from some height, say 1 m. Note to which it rises. This gives the velocities near the floor or ground, just before and just after the bounce (by using $v^2 = 2gh$ ). Hence you will get the coefficient of restitution.

Now take the big ball and a small ball and hold them in your hands one over the other, with the heavier ball below the lighter one, as shown here. Drop them together, taking care that they remain together while falling, and see what happens. You will find that the heavier ball rises less than when it was dropped alone, while the lighter one shoots up to about 3 m. With practice, you will be able to hold the ball properly so that the lighter ball rises vertically up and does not fly sideways. This is head-on collision.



You can try to find the best combination of balls which gives you the best effect. You can measure the masses on a standard balance. We leave it to you to think how you can determine the initial and final velocities of the balls.

$$= \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} v_{1i}^2$$

which is a positive quantity as expected.

Consider next an elastic collision. Using the above nomenclature with $\theta_1 = \theta_2 = 0$, the momentum and kinetic energy conservation equations are

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \qquad (6.24)$$

$$m_1 v_{1i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2 \qquad (6.25)$$

From Eqs. (6.24) and (6.25) it follows that,

$$m_1 v_{1i}(v_{2f} - v_{1i}) = m_1 v_{1f}(v_{2f} - v_{1f})$$

or, $$v_{2f}(v_{1i} - v_{1f}) = v_{1i}^2 - v_{1f}^2$$

$$= (v_{1i} - v_{1f})(v_{1i} + v_{1f})$$

Hence, $$\therefore v_{2f} = v_{1i} + v_{1f} \qquad (6.26)$$

Substituting this in Eq. (6.24), we obtain

$$v_{1f} = \frac{(m_1 - m_2)}{m_1 + m_2} v_{1i} \qquad (6.27)$$

and $$v_{2f} = \frac{2 m_1 v_{1i}}{m_1 + m_2} \qquad (6.28)$$

Thus, the 'unknowns' $\{v_{1f}, v_{2f}\}$ are obtained in terms of the 'knowns' $\{m_1, m_2, v_{1i}\}$. Special cases of our analysis are interesting.

**Case I :** If the two masses are equal

$$v_{1f} = 0$$
$$v_{2f} = v_{1i}$$

The first mass comes to rest and pushes off the second mass with its initial speed on collision.

**Case II :** If one mass dominates, e.g. $m_2 >> m_1$

$$v_{1f} \simeq -v_{1i} \qquad v_{2f} \simeq 0$$

The heavier mass is undisturbed while the lighter mass reverses its velocity.

▶ ***Example 6.12* Slowing down of neutrons**: In a nuclear reactor a neutron of high speed (typically $10^7$ m s$^{-1}$) must be slowed to $10^3$ m $s^{-1}$ so that it can have a high probability of interacting with isotope $^{235}_{92}U$ and causing it to fission. Show that a neutron can lose most of its kinetic energy in an elastic collision with a light nuclei like deuterium or carbon which has a mass of only a few times the neutron mass. The material making up the light nuclei, usually heavy water ($D_2O$) or graphite, is called a moderator.

*Answer* The initial kinetic energy of the neutron is

$$K_{1i} = \frac{1}{2} m_1 v_{1i}^2$$

while its final kinetic energy from Eq. (6.27)

$$K_{1f} = \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2} m_1 \left( \frac{m_1 - m_2}{m_1 + m_2} \right)^2 v_{1i}^2$$

The fractional kinetic energy lost is

$$f_1 = \frac{K_{1f}}{K_{1i}} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right)^2$$

while the fractional kinetic energy gained by the moderating nuclei $K_{2f}/K_{1i}$ is

$f_2 = 1 - f_1$ (elastic collision)

$$= \frac{4 m_1 m_2}{(m_1 + m_2)^2}$$

One can also verify this result by substituting from Eq. (6.28).

For deuterium $m_2 = 2m_1$ and we obtain $f_1 = 1/9$ while $f_2 = 8/9$. Almost 90% of the neutron's energy is transferred to deuterium. For carbon $f_1$ = 71.6% and $f_2$ = 28.4%. In practice, however, this number is smaller since head-on collisions are rare. ◀

If the initial velocities and final velocities of both the bodies are along the same straight line, then it is called a one-dimensional collision, or **head-on collision.** In the case of small spherical bodies, this is possible if the direction of travel of body 1 passes through the centre of body 2 which is at rest. In general, the collision is two-dimensional, where the initial velocities and the final velocities lie in a plane.

### 6.12.3 Collisions in Two Dimensions

Fig. 6.10 also depicts the collision of a moving mass $m_1$ with the stationary mass $m_2$. Linear momentum is conserved in such a collision. Since momentum is a vector this implies three equations for the three directions $\{x, y, z\}$. Consider the plane determined by the final velocity directions of $m_1$ and $m_2$ and choose it to be the $x$-$y$ plane. The conservation of the $z$-component of the linear momentum implies that the entire collision is in the $x$-$y$ plane. The $x$- and $y$-component equations are

$$m_1 v_{1i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2 \quad (6.29)$$

$$0 = m_1 v_{1f} \sin \theta_1 - m_2 v_{2f} \sin \theta_2 \quad (6.30)$$

One knows $\{m_1, m_2, v_{1i}\}$ in most situations. There are thus four unknowns $\{v_{1f}, v_{2f}, \theta_1 \text{ and } \theta_2\}$, and only two equations. If $\theta_1 = \theta_2 = 0$, we regain Eq. (6.24) for one dimensional collision.

If, further the collision is elastic,

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (6.31)$$

We obtain an additional equation. That still leaves us one equation short. At least one of the four unknowns, say $\theta_1$, must be made known for the problem to be solvable. For example, $\theta_1$ can be determined by moving a detector in an angular fashion from the $x$ to the $y$ axis. Given $\{m_1, m_2, v_{1i}, \theta_1\}$ we can determine $\{v_{1f}, v_{2f}, \theta_2\}$ from Eqs. (6.29)-(6.31).

▶ ***Example 6.13*** Consider the collision depicted in Fig. 6.10 to be between two billiard balls with equal masses $m_1 = m_2$. The first ball is called the cue while the second ball is called the target. The billiard player wants to 'sink' the target ball in a corner pocket, which is at an angle $\theta_2 = 37°$. Assume that the collision is elastic and that friction and rotational motion are not important. Obtain $\theta_1$.

*Answer* From momentum conservation, since the masses are equal

$$\mathbf{v}_{1i} = \mathbf{v}_{1f} + \mathbf{v}_{2f}$$

or

$$v_{1i}^2 = (\mathbf{v}_{1f} + \mathbf{v}_{2f}) \cdot (\mathbf{v}_{1f} + \mathbf{v}_{2f})$$

$$= v_{1f}^2 + v_{2f}^2 + 2 \mathbf{v}_{1f} \cdot \mathbf{v}_{2f}$$

$$= \quad v_{1f}^{\ 2} + v_{2f}^{\ 2} + 2v_{1f}v_{2f}\cos\ (\theta_1 + 37°)\ \} \qquad (6.32)$$

Since the collision is elastic and $m_1 = m_2$ it follows from conservation of kinetic energy that

$$v_{1i}^{\ 2} = v_{1f}^{\ 2} + v_{2f}^{\ 2} \qquad (6.33)$$

Comparing Eqs. (6.32) and (6.33), we get

$$\cos\ (\theta_1 + 37°) = 0$$

or $\quad \theta_1 + 37° = 90°$

Thus, $\quad \theta_1 = 53°$

This proves the following result : when two equal masses undergo a glancing elastic collision with one of them at rest, after the collision, they will move at right angles to each other. ◀

The matter simplifies greatly if we consider spherical masses with smooth surfaces, and assume that collision takes place only when the bodies touch each other. This is what happens in the games of marbles, carrom and billiards.

In our everyday world, collisions take place only when two bodies touch each other. But consider a comet coming from far distances to the sun, or alpha particle coming towards a nucleus and going away in some direction. Here we have to deal with forces involving action at a distance. Such an event is called scattering. The velocities and directions in which the two particles go away depend on their initial velocities as well as the type of interaction between them, their masses, shapes and sizes.

## SUMMARY

1. The *work-energy theorem* states that the change in kinetic energy of a body is the work done by the net force on the body.
$$K_f - K_i = W_{net}$$
2. A force is *conservative* if (i) work done by it on an object is path independent and depends only on the end points $\{x_i, x_j\}$, or (ii) the work done by the force is zero for an arbitrary closed path taken by the object such that it returns to its initial position.
3. For a conservative force in one dimension, we may define a *potential energy* function $V(x)$ such that
$$F(x) = -\frac{dV(x)}{dx}$$
or $$V_i - V_f = \int_{x_i}^{x_f} F(x)\,dx$$
4. The principle of conservation of mechanical energy states that the total mechanical energy of a body remains constant if the only forces that act on the body are conservative.
5. The *gravitational potential energy* of a particle of mass $m$ at a height $x$ about the earth's surface is
$$V(x) = m\,g\,x$$
where the variation of g with height is ignored.
6. The elastic potential energy of a spring of force constant $k$ and extension $x$ is
$$V(x) = \frac{1}{2}k\,x^2$$
7. The scalar or dot product of two vectors **A** and **B** is written as **A.B** and is a scalar quantity given by : **A·B** = $AB\cos\theta$, where $\theta$ is the angle between **A** and **B**. It can be positive, negative or zero depending upon the value of $\theta$. The scalar product of two vectors can be interpreted as the product of magnitude of one vector and component of the other vector along the first vector. For unit vectors :
$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1 \text{ and } \hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0$$
Scalar products obey the commutative and the distributive laws.