

Let $f(x) = \cos(\tan^{-1}(\sin(\cot^{-1}x)))$. The simplest form of $f(x)$ can be written as $\left(\frac{x^2 + A}{x^2 + B}\right)^{\frac{1}{2}}$. The value of $(A + B)$ is _____.

(1) 1

(2) 2

(3) 3

(4) 0

Answer (3)

Given $f(x) = \cos(\tan^{-1}(\sin(\cot^{-1}x)))$

$$= \cos\left(\tan^{-1}\left(\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right)\right)$$

$$= \cos\left(\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)$$

$$= \cos\left(\cos^{-1}\left(\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right)\right)$$

$$= \sqrt{\frac{1+x^2}{2+x^2}} = \left(\frac{x^2 + 1}{x^2 + 2}\right)^{\frac{1}{2}}$$

$\Rightarrow A = 1, B = 2$

$\Rightarrow A + B = 1 + 2 = 3$