If $\operatorname{cosec}^{-1} x + \cos^{-1} y + \sec^{-1} z \geq \lambda^2 - \sqrt{2\pi}\lambda + 3\pi$, *where* λ *is a real number, then*

(1) $x = 1, y = -1$

(2) $x = 1, z = -1$

(3) Both (1) & (2)

(4) $x = -1, z = -1$

Answer (3)

RHS $= \lambda^2 - \sqrt{2\pi}\lambda + 3\pi$

$= \lambda^2 - 2\sqrt{\frac{\pi}{2}}\lambda + \frac{\pi}{2} + 3\pi - \frac{\pi}{2}$

$= \left(\lambda - \frac{\pi}{2}\right)^2 + \frac{5\pi}{2} \geq \frac{5\pi}{2}$ ...(i)

$\because$ In LHS $\operatorname{cosec}^{-1}x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$, $\cos^{-1}y \in [0, \pi]$, $\sec^{-1}z \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\therefore$ LHS $\leq \frac{5\pi}{2}$ ...(ii)

From (i) and (ii) only possibility is sign of equality.

$\Rightarrow$ $x = 1, y = -1, z = -1$

Options (1) and (2) are correct.