The number of positive integers in the domain of $f(x) = \cos^{-1}([e^x] - 1) + \sin^{-1}([e^x])$, where $[\cdot]$ represents greatest integer function, is equal to 0

Sol. Let $[e^x] = t$ so, $g(t) = \cos^{-1}(t-1) + \sin^{-1}(t)$

for definition of $g(t)$;

$-1 \leq t-1 \leq 1 \Rightarrow 0 \leq t \leq 2$ —— (1)

and $-1 \leq t \leq 1 \Rightarrow -1 \leq t \leq 1$ —— (2)

from (1) & (2); $0 \leq t \leq 1$.

Since $t = [e^x] \Rightarrow 0 \leq [e^x] \leq 1$.

Since $[e^x]$ is an integer; it can have only two values; 0 and 1.

$\Rightarrow [e^x] = 0$ or $[e^x] = 1$

$\Rightarrow 0 \leq e^x < 1$ or $1 \leq e^x < 2$

$\Rightarrow 0 \leq e^x < 2$

$\Rightarrow -\infty < x < \ln 2$.

$\Rightarrow x \in (-\infty, \ln 2) = (-\infty, 0.693)$

$\therefore$ No. of positive integers in domain $= \boxed{0}$