Ques1.

Let $E_1 = \left\{x \in \mathbb{R} : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$

and $E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{ is a real number}\right\}$.

$\left(\text{Here, the inverse trigonometric function } \sin^{-1}x \text{ assumes values in } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].\right)$

Let $f : E_1 \to \mathbb{R}$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$

and $g : E_2 \to \mathbb{R}$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$.

LIST-I		LIST-II	
P.	The range of f is	**1.**	$\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
Q.	The range of g contains	**2.**	$(0, 1)$
R.	The domain of f contains	**3.**	$\left[-\frac{1}{2}, \frac{1}{2}\right]$
S.	The domain of g is	**4.**	$(-\infty, 0) \cup (0, \infty)$
		5.	$\left(-\infty, \frac{e}{e-1}\right]$
		6.	$(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is :

(A) **P → 4; Q → 2; R → 1; S → 1**

(B) **P → 3; Q → 3; R → 6; S → 5**

(C) **P → 4; Q → 2; R → 1; S → 6**

(D) **P → 4; Q → 3; R → 6; S → 5**

[JEE(Advanced)-2018]

Sol.

$E_1 : \dfrac{x}{x-1} > 0$

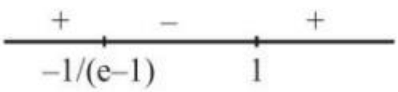
$\Rightarrow \quad E_1 : x \in (-\infty, 0)$ ¶$\cup$ $(1, \infty)$

$E_2 : -1 \le \ell n\left(\dfrac{x}{x+1}\right) \le 1$

$$\frac{1}{e} \le \frac{x}{x-1} \le e$$

Now $\dfrac{x}{x-1} - \dfrac{1}{e} \ge 0$

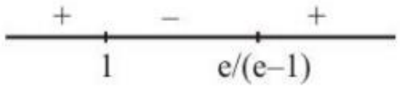
$$\Rightarrow \quad \frac{(e-1)x+1}{e(x-1)} \ge 0$$



$$\Rightarrow \quad x \in \left(-\infty, \frac{1}{1-e}\right] \cup (1, \infty)$$

also $\dfrac{x}{x-1} - e \le 0$

$$\frac{(e-1)x-e}{x-1} \geq 0$$



$$\Rightarrow \quad x \in (-\infty, 1) \cup \left[\frac{e}{e-1}, \infty\right]$$

$$\text{So} \quad E_2 : \left(-\infty, \frac{1}{1-e}\right) \cup \left[\frac{e}{e-1}, \infty\right]$$

as Range of $\frac{x}{x-1}$ is $R^{+} - \{1\}$

$\Rightarrow$ Range of f is $R - \{0\}$ or $(-\infty, 0) \cup (0, \infty)$

So, (A) is the correct option.