

**21.** If  $2 \sin^{-1} x + \{\cos^{-1} x\} > \frac{\pi}{2} + \{\sin^{-1} x\}$ , then  $x \in$ : (where  $\{\cdot\}$  denotes fractional part function)

(a)  $(\cos 1, 1]$

(b)  $[\sin 1, 1]$

(c)  $(\sin 1, 1]$

(d) None of these

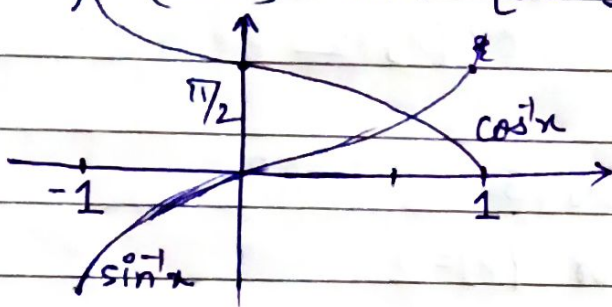
$$2 \sin^{-1} x + \{\cos^{-1} x\} > \frac{\pi}{2} + \{\sin^{-1} x\}$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} x + \cos^{-1} x - [\cos^{-1} x] > \frac{\pi}{2} + \sin^{-1} x - [\sin^{-1} x]$$

$$\Rightarrow \sin^{-1} x + \frac{\pi}{2} - [\cos^{-1} x] > \frac{\pi}{2} + \sin^{-1} x - [\sin^{-1} x]$$

$$\Rightarrow [\sin^{-1} x] > [\cos^{-1} x]$$

from the graphs of  $\sin^{-1} x$  and  $\cos^{-1} x$ , plot the graphs of  $[\sin^{-1} x]$  and  $[\cos^{-1} x]$ .



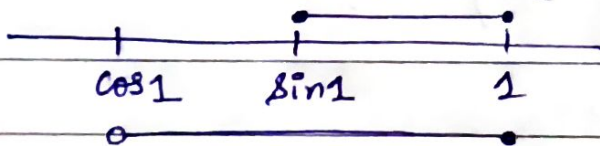
$$[\sin^{-1} x] = \begin{cases} -2 & -1 \leq x < -\sin 1 \\ -1 & -\sin 1 \leq x < 0 \\ 0 & 0 \leq x < \sin 1 \\ 1 & \sin 1 \leq x \leq 1 \end{cases}$$

$$[\cos^{-1} x] = \begin{cases} 3 & -1 \leq x \leq \cos 3 \\ 2 & \cos 3 < x \leq \cos 2 \\ 1 & \cos 2 < x \leq \cos 1 \\ 0 & \cos 1 < x \leq 1 \end{cases}$$

So,  $[\sin^{-1} x] > [\cos^{-1} x]$  is only

possible when  $[\sin^{-1} x] = 1$  &  $[\cos^{-1} x] = 0$

$$\therefore x \in [\sin 1, 1] \text{ \& } x \in [\cos 1, 1]$$



$\therefore x \in [\sin 1, 1]$  is the answer.

Note: This is a kind of deeply analytical question. Try to observe and make conclusions.