

32. The number of solutions of  $\sin^{-1} x + \sin^{-1}(1+x) = \cos^{-1} x$  is/are :

(a) 0

(b) 1

(c) 2

(d) infinite

$\sin^{-1}(1+x)$  is defined for  $x < 0$  and  $\sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x \forall -1 \leq x \leq 1$ .

The given equation is  $\sin^{-1} x + \sin^{-1}(1+x) = \cos^{-1} x$

which can be written as

$$\frac{\pi}{2} - \cos^{-1} x + \frac{\pi}{2} - \cos^{-1}(1+x) = \cos^{-1} x$$

$$\Rightarrow \pi - \cos^{-1}(1+x) = 2 \cos^{-1} x$$

$$\Rightarrow \cos^{-1}(-1-x) = 2\pi - \cos^{-1}(2x^2 - 1)$$

$$\Rightarrow \cos^{-1}(-1-x) + \cos^{-1}(2x^2 - 1) = 2\pi$$

$$\Rightarrow \cos^{-1}(-1-x) = \cos^{-1}(2x^2 - 1) = \pi$$

$$\Rightarrow -1-x = 2x^2 - 1 = -1$$

$$\Rightarrow x = 0$$

which implies that the total number of solutions  $\sin^{-1} x + \sin^{-1}(1+x) = \cos^{-1} x$  is only one.