

45. Range of $f(x) = \sin^{-1}[x - 1] + 2 \cos^{-1}[x - 2]$ ($[\cdot]$ denotes greatest integer function)

(a) $\left\{-\frac{\pi}{2}, 0\right\}$

(b) $\left\{\frac{\pi}{2}, 2\pi\right\}$

(c) $\left\{\frac{\pi}{4}, \frac{\pi}{2}\right\}$

(d) $\left\{\frac{3\pi}{2}, 2\pi\right\}$

②.

$$f(x) = \sin^{-1}[x-1] + 2\cos^{-1}[x-2]$$

for definition of $f(x)$; $-1 \leq [x]-1 \leq 1$
 $\Rightarrow 0 \leq [x] \leq 2$ — (1)

Also, $-1 \leq [x]-2 \leq 1$

\Rightarrow ~~0~~ $1 \leq [x] \leq 3$ — (2)

from (1) & (2); $[x]=1$ or $[x]=2$

\therefore Range has two elements.

If $[x]=1$, $\sin^{-1}(1-1) + 2\cos^{-1}(1-2) = 2\pi$

If $[x]=2$; $\sin^{-1}(2-1) + 2\cos^{-1}(2-2) = 3\pi/2$

$\therefore R = \{3\pi/2, 2\pi\}$