dominates over that due to ray optics (i.e., the size a of the aperture). Equation (10.31) also shows that ray optics is valid in the limit of wavelength tending to zero.

EXAMPLE 10.7

Example 10.7 For what distance is ray optics a good approximation when the aperture is 3 mm wide and the wavelength is 500 nm?

Solution $z_F = \dfrac{a^2}{\lambda} = \dfrac{\left(3 \times 10^{-3}\right)^2}{5 \times 10^{-7}} = 18 \text{ m}$

This example shows that even with a small aperture, diffraction spreading can be neglected for rays many metres in length. Thus, ray optics is valid in many common situations.

10.7 POLARISATION

Consider holding a long string that is held horizontally, the other end of which is assumed to be fixed. If we move the end of the string up and down in a periodic manner, we will generate a wave propagating in the +x direction (Fig. 10.21). Such a wave could be described by the following equation

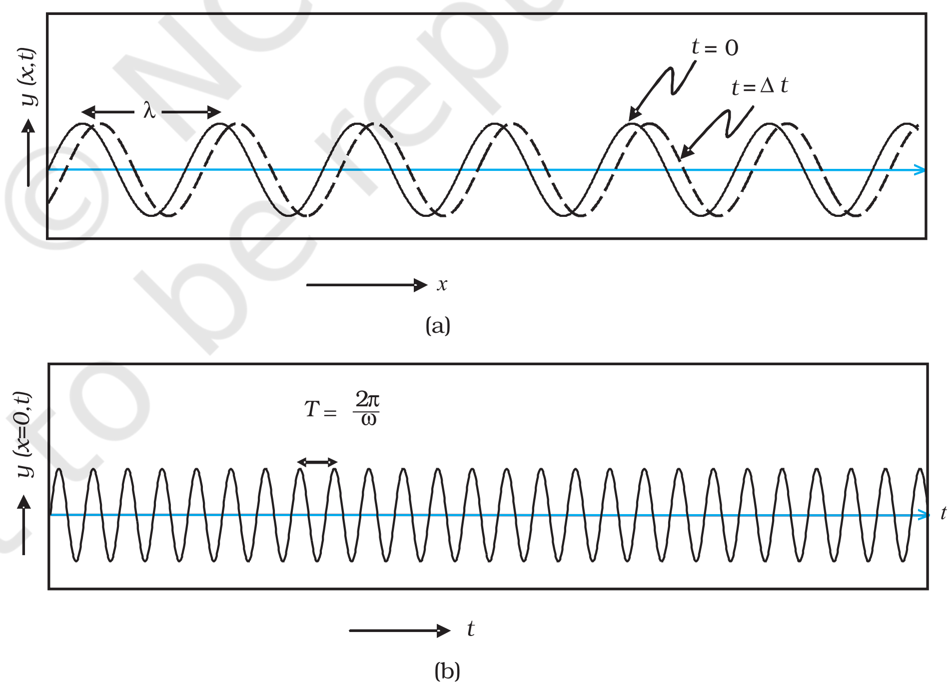


FIGURE 10.21 (a) The curves represent the displacement of a string at t = 0 and at $t = \Delta t$, respectively when a sinusoidal wave is propagating in the +x-direction. (b) The curve represents the time variation of the displacement at x = 0 when a sinusoidal wave is propagating in the +x-direction. At $x = \Delta x$, the time variation of the displacement will be slightly displaced to the right.

$$y(x,t) = a \sin(kx - \omega t) \tag{10.32}$$

where a and $\omega (= 2\pi\nu)$ represent the amplitude and the angular frequency of the wave, respectively; further,

$$\lambda = \frac{2\pi}{k} \tag{10.33}$$

represents the wavelength associated with the wave. We had discussed propagation of such waves in Chapter 15 of Class XI textbook. Since the displacement (which is along the y direction) is at right angles to the direction of propagation of the wave, we have what is known as a *transverse wave*. Also, since the displacement is in the y direction, it is often referred to as a y-polarised wave. Since each point on the string moves on a straight line, the wave is also referred to as a linearly polarised wave. Further, the string always remains confined to the x-y plane and therefore it is also referred to as a *plane polarised wave*.

In a similar manner we can consider the vibration of the string in the x-z plane generating a z-polarised wave whose displacement will be given by

$$z(x,t) = a \sin(kx - \omega t) \tag{10.34}$$

It should be mentioned that the linearly polarised waves [described by Eqs. (10.32) and (10.34)] are all transverse waves; i.e., the displacement of each point of the string is always at right angles to the direction of propagation of the wave. Finally, if the plane of vibration of the string is changed randomly in very short intervals of time, then we have what is known as an *unpolarised wave*. Thus, for an unpolarised wave the displacement will be randomly changing with time though it will always be perpendicular to the direction of propagation.

Light waves are transverse in nature; i.e., the electric field associated with a propagating light wave is always at right angles to the direction of propagation of the wave. This can be easily demonstrated using a simple polaroid. You must have seen thin plastic like sheets, which are called *polaroids*. A polaroid consists of long chain molecules aligned in a particular direction. The electric vectors (associated with the propagating light wave) along the direction of the aligned molecules get absorbed. Thus, if an unpolarised light wave is incident on such a polaroid then the light wave will get linearly polarised with the electric vector oscillating along a direction perpendicular to the aligned molecules; this direction is known as the *pass-axis* of the polaroid.

Thus, if the light from an ordinary source (like a sodium lamp) passes through a polaroid sheet P_1, it is observed that its intensity is reduced by half. Rotating P_1 has no effect on the transmitted beam and transmitted intensity remains constant. Now, let an identical piece of polaroid P_2 be placed before P_1. As expected, the light from the lamp is reduced in intensity on passing through P_2 alone. But now rotating P_1 has a dramatic effect on the light coming from P_2. In one position, the intensity transmitted by P_2 followed by P_1 is nearly zero. When turned by 90° from this position, P_1 transmits nearly the full intensity emerging from P_2 (Fig. 10.22).

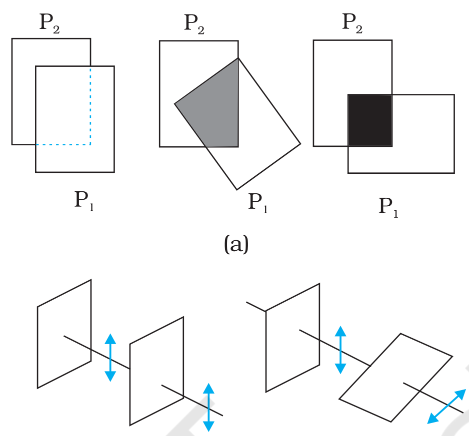


FIGURE 10.22 (a) Passage of light through two polaroids P_2 and P_1. The transmitted fraction falls from 1 to 0 as the angle between them varies from 0° to 90°. Notice that the light seen through a single polaroid P_1 does not vary with angle. (b) Behaviour of the electric vector when light passes through two polaroids. The transmitted polarisation is the component parallel to the polaroid axis. The double arrows show the oscillations of the electric vector.

The above experiment can be easily understood by assuming that light passing through the polaroid P_2 gets polarised along the pass-axis of P_2. If the pass-axis of P_2 makes an angle θ with the pass-axis of P_1, then when the polarised beam passes through the polaroid P_2, the component $E \cos \theta$ (along the pass-axis of P_2) will pass through P_2. Thus, as we rotate the polaroid P_1 (or P_2), the intensity will vary as:

$$I = I_0 \cos^2\theta \tag{10.35}$$

where I_0 is the intensity of the polarized light after passing through P_1. This is known as *Malus' law*. The above discussion shows that the intensity coming out of a single polaroid is half of the incident intensity. By putting a second polaroid, the intensity can be further controlled from 50% to zero of the incident intensity by adjusting the angle between the pass-axes of two polaroids.

Polaroids can be used to control the intensity, in sunglasses, windowpanes, etc. Polaroids are also used in photographic cameras and 3D movie cameras.

EXAMPLE 10.8

Example 10.8 Discuss the intensity of transmitted light when a polaroid sheet is rotated between two crossed polaroids?

Solution Let I_0 be the intensity of polarised light after passing through the first polariser P_1. Then the intensity of light after passing through second polariser P_2 will be

$I = I_0 \cos^2\theta$,

where θ is the angle between pass axes of P_1 and P_2. Since P_1 and P_3 are crossed the angle between the pass axes of P_2 and P_3 will be $(\pi/2-\theta)$. Hence the intensity of light emerging from P_3 will be

$$I = I_0 \cos^2\theta \cos^2\left(\frac{\pi}{2}-\theta\right)$$

$$= I_0 \cos^2\theta \sin^2\theta = (I_0/4)\sin^2 2\theta$$

Therefore, the transmitted intensity will be maximum when $\theta = \pi/4$.

EXAMPLE 10.8

10.7.1 Polarisation by scattering

The light from a clear blue portion of the sky shows a rise and fall of intensity when viewed through a polaroid which is rotated. This is nothing but sunlight, which has changed its direction (having been scattered) on encountering the molecules of the earth's atmosphere. As Fig. 10.23(a) shows, the incident sunlight is unpolarised. The dots stand for polarisation perpendicular to the plane of the figure. The double arrows show polarisation in the plane of the figure. (There is no phase relation between these two in unpolarised light). Under the influence of the electric field of the incident wave the electrons in the molecules acquire components of motion in both these directions. We have drawn an observer looking at 90° to the direction of the sun. Clearly, charges accelerating parallel to the double arrows do not radiate energy towards this observer since their acceleration has no transverse component. The radiation scattered by the molecule is therefore represented by dots. It is polarised perpendicular to the plane of the figure. This explains the polarisation of scattered light from the sky.

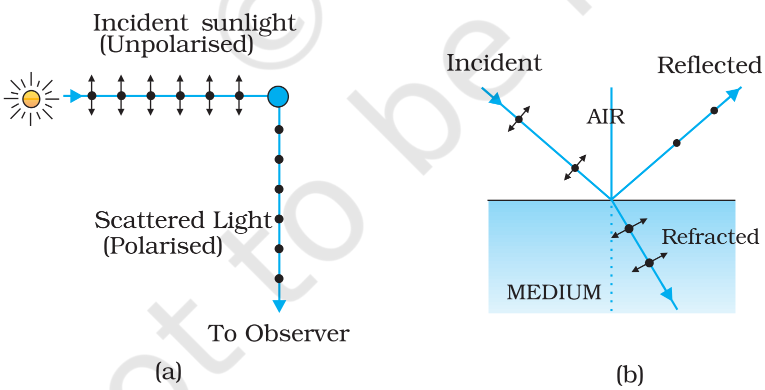
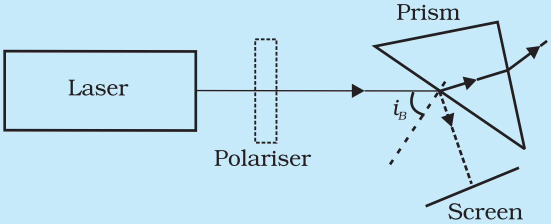


FIGURE 10.23 (a) Polarisation of the blue scattered light from the sky. The incident sunlight is unpolarised (dots and arrows). A typical molecule is shown. It scatters light by 90° polarised normal to the plane of the paper (dots only). (b) Polarisation of light reflected from a transparent medium at the Brewster angle (reflected ray perpendicular to refracted ray).

The scattering of light by molecules was intensively investigated by C.V. Raman and his collaborators in Kolkata in the 1920s. Raman was awarded the Nobel Prize for Physics in 1930 for this work.

A SPECIAL CASE OF TOTAL TRANSMISSION

When light is incident on an interface of two media, it is observed that some part of it gets reflected and some part gets transmitted. Consider a related question: *Is it possible that under some conditions a monochromatic beam of light incident on a surface (which is normally reflective) gets completely transmitted with no reflection?* To your surprise, the answer is *yes*.



Let us try a simple experiment and check what happens. Arrange a laser, a good polariser, a prism and screen as shown in the figure here.

Let the light emitted by the laser source pass through the polariser and be incident on the surface of the prism at the Brewster's angle of incidence i_B. Now rotate the polariser carefully and you will observe that for a specific alignment of the polariser, the light incident on the prism is completely transmitted and no light is reflected from the surface of the prism. The reflected spot will completely vanish.

10.7.2 Polarisation by reflection

Figure 10.23(b) shows light reflected from a transparent medium, say, water. As before, the dots and arrows indicate that both polarisations are present in the incident and refracted waves. We have drawn a situation in which the reflected wave travels at right angles to the refracted wave. The oscillating electrons in the water produce the reflected wave. These move in the two directions transverse to the radiation from wave in the medium, i.e., the *refracted wave*. The arrows are parallel to the direction of the *reflected* wave. Motion in this direction does not contribute to the reflected wave. As the figure shows, the reflected light is therefore linearly polarised perpendicular to the plane of the figure (represented by dots). This can be checked by looking at the reflected light through an analyser. The transmitted intensity will be zero when the axis of the analyser is in the plane of the figure, i.e., the plane of incidence.

When unpolarised light is incident on the boundary between two transparent media, the reflected light is polarised with its electric vector perpendicular to the plane of incidence when the refracted and reflected rays make a right angle with each other. Thus we have seen that when reflected wave is perpendicular to the refracted wave, the reflected wave is a totally polarised wave. The angle of incidence in this case is called *Brewster's angle* and is denoted by i_B. We can see that i_B is related to the refractive index of the denser medium. Since we have $i_B + r = \pi/2$, we get from Snell's law

$$\mu = \frac{\sin i_B}{\sin r} = \frac{\sin i_B}{\sin(\pi/2 - i_B)}$$

$$= \frac{\sin i_B}{\cos i_B} = \tan i_B \tag{10.36}$$

This is known as *Brewster's law*.

Example 10.9 Unpolarised light is incident on a plane glass surface. What should be the angle of incidence so that the reflected and refracted rays are perpendicular to each other?

Solution For $i + r$ to be equal to $\pi/2$, we should have $\tan i_B = \mu = 1.5$. This gives $i_B = 57°$. This is the Brewster's angle for air to glass interface.

EXAMPLE 10.9

For simplicity, we have discussed scattering of light by 90°, and reflection at the Brewster angle. In this special situation, one of the two perpendicular components of the electric field is zero. At other angles, both components are present but one is stronger than the other. There is no stable phase relationship between the two perpendicular components since these are derived from two perpendicular components of an unpolarised beam. When such light is viewed through a rotating analyser, one sees a maximum and a minimum of intensity but not complete darkness. This kind of light is called *partially polarised*.

Let us try to understand the situation. When an unpolarised beam of light is incident at the Brewster's angle on an interface of two media, only part of light with electric field vector perpendicular to the plane of incidence will be reflected. Now by using a good polariser, if we completely remove all the light with its electric vector perpendicular to the plane of incidence and let this light be incident on the surface of the prism at Brewster's angle, you will then observe no reflection and there will be total transmission of light.

We began this chapter by pointing out that there are some phenomena which can be explained only by the wave theory. In order to develop a proper understanding, we first described how some phenomena like reflection and refraction, which were studied on this basis of Ray Optics in Chapter 9, can also be understood on the basis of Wave Optics. Then we described Young's double slit experiment which was a turning point in the study of optics. Finally, we described some associated points such as diffraction, resolution, polarisation, and validity of ray optics. In the next chapter, you will see how new experiments led to new theories at the turn of the century around 1900 A.D.

SUMMARY

1. Huygens' principle tells us that each point on a wavefront is a source of secondary waves, which add up to give the wavefront at a later time.
2. Huygens' construction tells us that the new wavefront is the forward envelope of the secondary waves. When the speed of light is independent of direction, the secondary waves are spherical. The rays are then perpendicular to both the wavefronts and the time of travel