(b) To enable them to conduct charge (produced by friction) to the ground; as too much of static electricity accumulated may result in spark and result in fire.

(c) Reason similar to (b).

(d) Current passes only when there is difference in potential.

EXAMPLE 2.7

2.10 Dielectrics and Polarisation

Dielectrics are non-conducting substances. In contrast to conductors, they have no (or negligible number of) charge carriers. Recall from Section 2.9 what happens when a conductor is placed in an external electric field. The free charge carriers move and charge distribution in the conductor adjusts itself in such a way that the electric field due to induced charges opposes the external field within the conductor. This happens until, in the static situation, the two fields cancel each other and the net electrostatic field in the conductor is zero. In a dielectric, this free movement of charges is not possible. It turns out that the external field induces dipole moment by stretching or re-orienting molecules of the dielectric. The collective effect of all the molecular dipole moments is net charges on the surface of the dielectric which produce a field that opposes the external field. Unlike in a conductor, however, the opposing field so induced does not exactly cancel the external field. It only reduces it. The extent of the effect depends on the nature of the dielectric. To understand the effect, we need to look at the charge distribution of a dielectric at the molecular level.

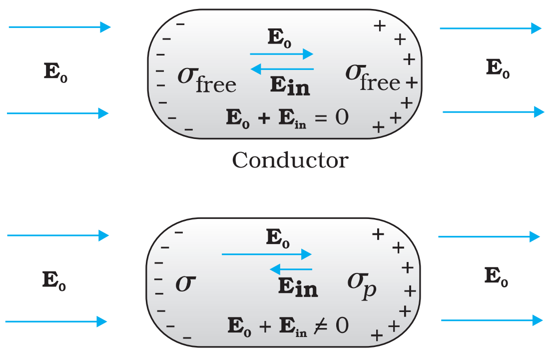


FIGURE 2.20 Difference in behaviour of a conductor and a dielectric in an external electric field.

The molecules of a substance may be polar or non-polar. In a non-polar molecule, the centres of positive and negative charges coincide. The molecule then has no permanent (or intrinsic) dipole moment. Examples of non-polar molecules are oxygen (O_2) and hydrogen (H_2) molecules which, because of their symmetry, have no dipole moment. On the other hand, a polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field). Such molecules have a permanent dipole moment. An ionic molecule such as HCl or a molecule of water (H_2O) are examples of polar molecules.

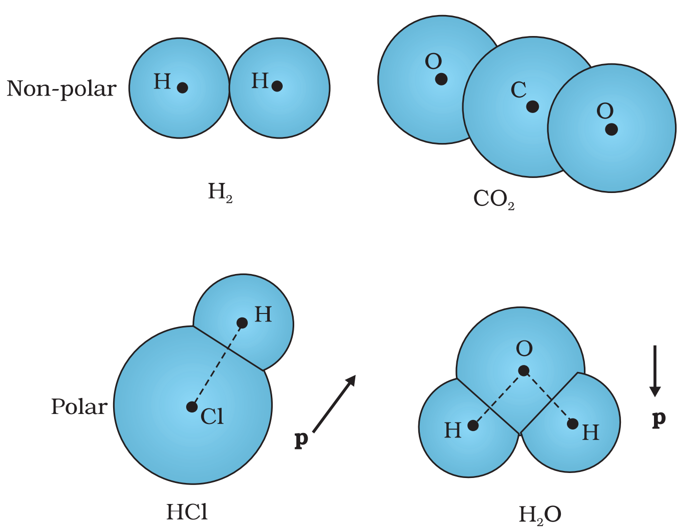
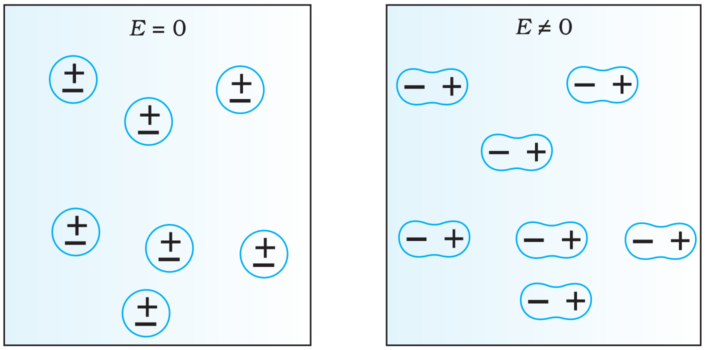
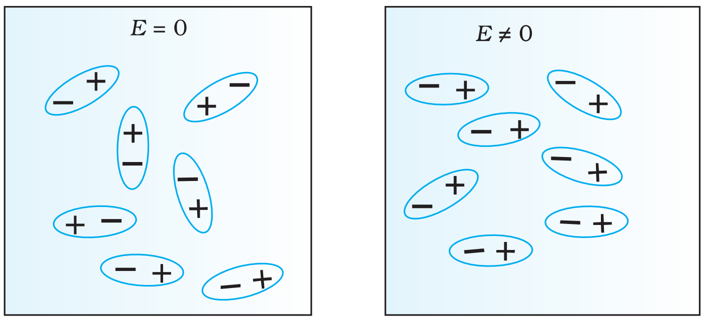


FIGURE 2.21 Some examples of polar and non-polar molecules.



(a) Non-polar molecules



(b) Polar molecules

FIGURE 2.22 A dielectric develops a net dipole moment in an external electric field. (a) Non-polar molecules, (b) Polar molecules.

In an external electric field, the positive and negative charges of a non-polar molecule are displaced in opposite directions. The displacement stops when the external force on the constituent charges of the molecule is balanced by the restoring force (due to internal fields in the molecule). The non-polar molecule thus develops an induced dipole moment. The dielectric is said to be polarised by the external field. We consider only the simple situation when the induced dipole moment is in the direction of the field and is proportional to the field strength. (Substances for which this assumption is true are called *linear isotropic dielectrics*.) The induced dipole moments of different molecules add up giving a net dipole moment of the dielectric in the presence of the external field.

A dielectric with polar molecules also develops a net dipole moment in an external field, but for a different reason. In the absence of any external field, the different permanent dipoles are oriented randomly due to thermal agitation; so the total dipole moment is zero. When an external field is applied, the individual dipole moments tend to align with the field. When summed overall the molecules, there is then a net dipole moment in the direction of the external field, i.e., the dielectric is polarised. The extent of polarisation depends on the relative strength of two mutually opposite factors: the dipole potential energy in the external field tending to align the dipoles with the field and thermal energy tending to disrupt the alignment. There may be, in addition, the 'induced dipole moment' effect as for non-polar molecules, but generally the alignment effect is more important for polar molecules.

Thus in either case, whether polar or non-polar, a dielectric develops a net dipole moment in the presence of an external field. The dipole moment per unit volume is called *polarisation* and is denoted by **P**. For linear isotropic dielectrics,

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E} \qquad (2.37)$$

where χ_e is a constant characteristic of the dielectric and is known as the *electric susceptibility* of the dielectric medium.

It is possible to relate χ_e to the molecular properties of the substance, but we shall not pursue that here.

The question is: how does the polarised dielectric modify the original external field inside it? Let us consider, for simplicity, a rectangular dielectric slab placed in a uniform external field $\mathbf{E}_0$ parallel to two of its faces. The field causes a uniform polarisation **P** of the dielectric. Thus every volume element Δv of the slab has a dipole moment $\mathbf{P}\,\Delta v$ in the direction of the field. The volume element Δv is macroscopically small but contains a very large number of molecular dipoles. Anywhere inside the dielectric, the volume element Δv has no net charge (though it has net dipole moment). This is, because, the positive charge of one dipole sits close to the negative charge of the adjacent dipole. However, at the surfaces of the dielectric normal to the electric field, there is evidently a net charge density. As seen in Fig 2.23, the positive ends of the dipoles remain unneutralised at the right surface and the negative ends at the left surface. The unbalanced charges are the induced charges due to the external field.

Thus, the polarised dielectric is equivalent to two charged surfaces with induced surface charge densities, say σ_p and $-\sigma_p$. Clearly, the field produced by these surface charges opposes the external field. The total field in the dielectric is, thereby, reduced from the case when no dielectric is present. We should note that the surface charge density $\pm\sigma_p$ arises from bound (not free charges) in the dielectric.

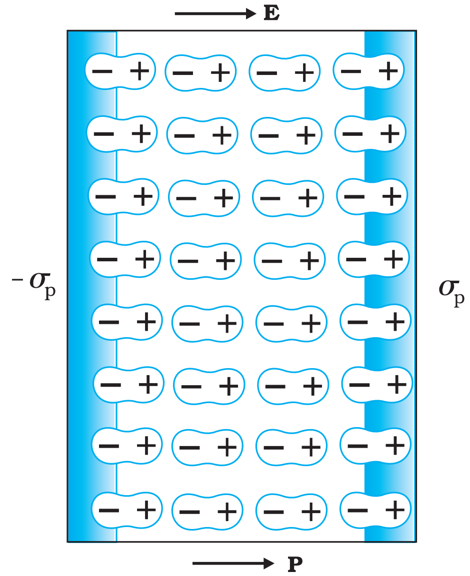


FIGURE 2.23 A uniformly polarised dielectric amounts to induced surface charge density, but no volume charge density.

2.11 Capacitors and Capacitance

A capacitor is a system of two conductors separated by an insulator (Fig. 2.24). The conductors have charges, say Q_1 and Q_2, and potentials V_1 and V_2. Usually, in practice, the two conductors have charges Q and $-Q$, with potential difference $V = V_1 - V_2$ between them. We shall consider only this kind of charge configuration of the capacitor. (Even a single conductor can be used as a capacitor by assuming the other at infinity.) The conductors may be so charged by connecting them to the two terminals of a battery. Q is called the charge of the capacitor, though this, in fact, is the charge on one of the conductors – the total charge of the capacitor is zero.

The electric field in the region between the conductors is proportional to the charge Q. That is, if the charge on the capacitor is, say doubled, the electric field will also be doubled at every point. (This follows from the direct proportionality between field and charge implied by Coulomb's law and the superposition principle.) Now, potential difference V is the work done per unit positive charge in taking a small test charge from the conductor 2 to 1 against the field. Consequently, V is also proportional to Q, and the ratio Q/V is a constant:

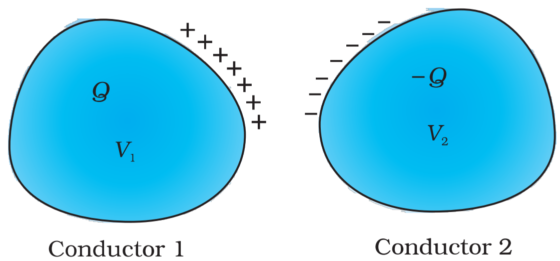


FIGURE 2.24 A system of two conductors separated by an insulator forms a capacitor.

$$C = \frac{Q}{V} \tag{2.38}$$

The constant C is called the *capacitance* of the capacitor. C is independent of Q or V, as stated above. The capacitance C depends only on the geometrical configuration (shape, size, separation) of the system of two conductors. [As we shall see later, it also depends on the nature of the insulator (dielectric) separating the two conductors.] The SI unit of capacitance is 1 farad (=1 coulomb volt^{-1}) or 1 F = 1 C V^{-1}. A capacitor with fixed capacitance is symbolically shown as ⊣ ⊢, while the one with variable capacitance is shown as ⫲.

Equation (2.38) shows that for large C, V is small for a given Q. This means a capacitor with large capacitance can hold large amount of charge Q at a relatively small V. This is of practical importance. High potential difference implies strong electric field around the conductors. A strong electric field can ionise the surrounding air and accelerate the charges so produced to the oppositely charged plates, thereby neutralising the charge on the capacitor plates, at least partly. In other words, the charge of the capacitor leaks away due to the reduction in insulating power of the intervening medium.

The maximum electric field that a dielectric medium can withstand without break-down (of its insulating property) is called its *dielectric strength*; for air it is about 3×10^6 Vm^{-1}. For a separation between conductors of the order of 1 cm or so, this field corresponds to a potential difference of 3×10^4 V between the conductors. Thus, for a capacitor to store a large amount of charge without leaking, its capacitance should be high enough so that the potential difference and hence the electric field do not exceed the break-down limits. Put differently, there is a limit to the amount of charge that can be stored on a given capacitor without significant leaking. In practice, a farad is a very big unit; the most common units are its sub-multiples 1 μF = 10^{-6} F, 1 nF = 10^{-9} F, 1 pF = 10^{-12} F, etc. Besides its use in storing charge, a capacitor is a key element of most ac circuits with important functions, as described in Chapter 7.

2.12 The Parallel Plate Capacitor

A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance (Fig. 2.25). We first take the intervening medium between the plates to be vacuum. The effect of a dielectric medium between the plates is discussed in the next section. Let A be the area of each plate and d the separation between them. The two plates have charges Q and $-Q$. Since d is much smaller than the linear dimension of the plates ($d^2 << A$), we can use the result on electric field by an infinite plane sheet of uniform surface charge density (Section 1.15). Plate 1 has surface charge density $\sigma = Q/A$ and plate 2 has a surface charge density $-\sigma$. Using Eq. (1.33), the electric field in different regions is:

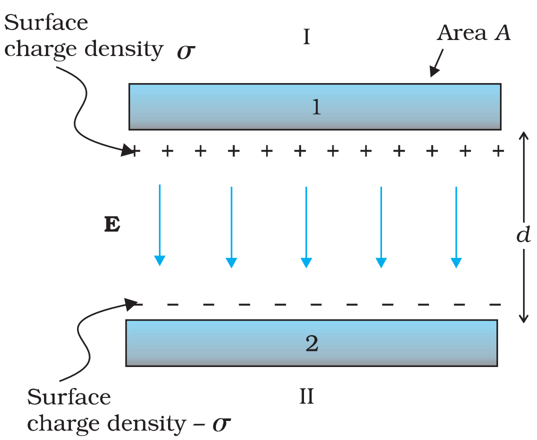


FIGURE 2.25 The parallel plate capacitor.

Outer region I (region above the plate 1),

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0 \qquad (2.39)$$

Outer region II (region below the plate 2),

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0 \tag{2.40}$$

In the inner region between the plates 1 and 2, the electric fields due to the two charged plates add up, giving

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \tag{2.41}$$

The direction of electric field is from the positive to the negative plate.

Thus, the electric field is localised between the two plates and is uniform throughout. For plates with finite area, this will not be true near the outer boundaries of the plates. The field lines bend outward at the edges — an effect called 'fringing of the field'. By the same token, σ will not be strictly uniform on the entire plate. [E and σ are related by Eq. (2.35).] However, for $d^2 << A$, these effects can be ignored in the regions sufficiently far from the edges, and the field there is given by Eq. (2.41). Now for uniform electric field, potential difference is simply the electric field times the distance between the plates, that is,

$$V = E\,d = \frac{1}{\varepsilon_0}\frac{Qd}{A} \tag{2.42}$$

The capacitance C of the parallel plate capacitor is then

$$C = \frac{Q}{V} = = \frac{\varepsilon_0 A}{d} \tag{2.43}$$

which, as expected, depends only on the geometry of the system. For typical values like A = 1 m^2, d = 1 mm, we get

$$C = \frac{8.85 \times 10^{-12} C^2 N^{-1} m^{-2} \times 1 m^2}{10^{-3} m} = 8.85 \times 10^{-9} F \tag{2.44}$$

(You can check that if 1F= 1C V^{-1} = 1C (NC^{-1}m)$^{-1}$ = 1 C^2 N^{-1}m^{-1}.) This shows that 1F is too big a unit in practice, as remarked earlier. Another way of seeing the 'bigness' of 1F is to calculate the area of the plates needed to have C = 1F for a separation of, say 1 cm:

$$A = \frac{Cd}{\varepsilon_0} = \frac{1F \times 10^{-2} m}{8.85 \times 10^{-12} C^2 N^{-1} m^{-2}} = 10^9 m^2 \tag{2.45}$$

which is a plate about 30 km in length and breadth!

Factors affecting capacitance, capacitors in action
Interactive Java tutorial
http://micro.magnet.fsu.edu/electromag/java/capacitance/

2.13 Effect of Dielectric on Capacitance

With the understanding of the behaviour of dielectrics in an external field developed in Section 2.10, let us see how the capacitance of a parallel plate capacitor is modified when a dielectric is present. As before, we have two large plates, each of area A, separated by a distance d. The charge on the plates is $\pm Q$, corresponding to the charge density $\pm\sigma$ (with $\sigma = Q/A$). When there is vacuum between the plates,

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

and the potential difference V_0 is

$V_0 = E_0 d$

The capacitance C_0 in this case is

$$C_0 = \frac{Q}{V_0} = \varepsilon_0 \frac{A}{d} \tag{2.46}$$

Consider next a dielectric inserted between the plates fully occupying the intervening region. The dielectric is polarised by the field and, as explained in Section 2.10, the effect is equivalent to two charged sheets (at the surfaces of the dielectric normal to the field) with surface charge densities σ_p and $-\sigma_p$. The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is $\pm(\sigma - \sigma_p)$. That is,

$$E = \frac{\sigma - \sigma_P}{\varepsilon_0} \tag{2.47}$$

so that the potential difference across the plates is

$$V = E\,d = \frac{\sigma - \sigma_P}{\varepsilon_0} d \tag{2.48}$$

For linear dielectrics, we expect σ_p to be proportional to E_0, i.e., to σ. Thus, $(\sigma - \sigma_p)$ is proportional to σ and we can write

$$\sigma - \sigma_P = \frac{\sigma}{K} \tag{2.49}$$

where K is a constant characteristic of the dielectric. Clearly, $K > 1$. We then have

$$V = \frac{\sigma d}{\varepsilon_0 K} = \frac{Qd}{A\varepsilon_0 K} \tag{2.50}$$

The capacitance C, with dielectric between the plates, is then

$$C = \frac{Q}{V} = \frac{\varepsilon_0 KA}{d} \tag{2.51}$$

The product $\varepsilon_0 K$ is called the *permittivity* of the medium and is denoted by ε

$$\varepsilon = \varepsilon_0 K \tag{2.52}$$

For vacuum $K = 1$ and $\varepsilon = \varepsilon_0$; ε_0 is called the *permittivity of the vacuum*. The dimensionless ratio

$$K = \frac{\varepsilon}{\varepsilon_0} \tag{2.53}$$

is called the *dielectric constant* of the substance. As remarked before, from Eq. (2.49), it is clear that K is greater than 1. From Eqs. (2.46) and (2. 51)

$$K = \frac{C}{C_0} \tag{2.54}$$

Thus, the dielectric constant of a substance is the factor (>1) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor. Though we arrived at

Eq. (2.54) for the case of a parallel plate capacitor, it holds good for any type of capacitor and can, in fact, be viewed in general as a definition of the dielectric constant of a substance.

ELECTRIC DISPLACEMENT

We have introduced the notion of dielectric constant and arrived at Eq. (2.54), without giving the explicit relation between the induced charge density σ_p and the polarisation **P**.

We take without proof the result that

$$\sigma_P = \mathbf{P} \cdot \hat{\mathbf{n}}$$

where $\hat{\mathbf{n}}$ is a unit vector along the outward normal to the surface. Above equation is general, true for any shape of the dielectric. For the slab in Fig. 2.23, **P** is along $\hat{\mathbf{n}}$ at the right surface and opposite to $\hat{\mathbf{n}}$ at the left surface. Thus at the right surface, induced charge density is positive and at the left surface, it is negative, as guessed already in our qualitative discussion before. Putting the equation for electric field in vector form

$$\mathbf{E} \cdot \hat{\mathbf{n}} = \frac{\sigma - \mathbf{P} \cdot \hat{\mathbf{n}}}{\varepsilon_0}$$

or $(\varepsilon_0 \mathbf{E} + \mathbf{P}) \cdot \hat{\mathbf{n}} = \sigma$

The quantity $\varepsilon_0 \mathbf{E} + \mathbf{P}$ is called the *electric displacement* and is denoted by **D**. It is a vector quantity. Thus,

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \ \mathbf{D} \cdot \hat{\mathbf{n}} = \sigma,$$

The significance of **D** is this : in vacuum, **E** is related to the free charge density σ. When a dielectric medium is present, the corresponding role is taken up by **D**. For a dielectric medium, it is **D** not **E** that is directly related to free charge density σ, as seen in above equation. Since **P** is in the same direction as **E**, all the three vectors **P**, **E** and **D** are parallel.

The ratio of the magnitudes of **D** and **E** is

$$\frac{D}{E} = \frac{\sigma \varepsilon_0}{\sigma - \sigma_P} = \varepsilon_0 K$$

Thus,

$$\mathbf{D} = \varepsilon_0 K \mathbf{E}$$

and $\mathbf{P} = \mathbf{D} - \varepsilon_0 \mathbf{E} = \varepsilon_0 (K-1)\mathbf{E}$

This gives for the electric susceptibility χ_e defined in Eq. (2.37)

$\chi_e = (K-1)$

Example 2.8 A slab of material of dielectric constant K has the same area as the plates of a parallel-plate capacitor but has a thickness $(3/4)d$, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates?

Solution Let $E_0 = V_0/d$ be the electric field between the plates when there is no dielectric and the potential difference is V_0. If the dielectric is now inserted, the electric field in the dielectric will be $E = E_0/K$. The potential difference will then be

EXAMPLE 2.8