4) Prove that the product of length of perpendicular(s) drawn from the points $(ae, 0)$ & $(-ae, 0)$ to the line

$$y = mx + \sqrt{a^2m^2+b^2} \quad \text{is equal to } b^2.$$

[Note: $a, b, m, e \in \mathbb{R}$ and $\boxed{b^2 = a^2(1-e^2)}$ given.

Soln

⊥r dist of $(ae, 0)$ from $y = mx + \sqrt{a^2m^2+b^2}$ is

$$d_1 = \left|\left(\frac{mae + \sqrt{a^2m^2+b^2}}{\sqrt{m^2+1}}\right)\right|$$

⊥r dist of $(-ae, 0)$ from $y = mx + \sqrt{a^2m^2+b^2}$ is

$$d_2 = \left|\left(\frac{-mae + \sqrt{a^2m^2+b^2}}{\sqrt{m^2+1}}\right)\right|$$

Product

$$d_1d_2 = \left|\frac{(\sqrt{a^2m^2+b^2} - mae)(\sqrt{a^2m^2+b^2} + mae)}{(\sqrt{m^2+1})^2}\right|$$

$$d_1d_2 = \left|\frac{a^2m^2 + b^2 - m^2a^2e^2}{m^2+1}\right| \qquad \boxed{\text{Since } b^2 = -a^2e^2 + a^2}$$

$$= \frac{b^2 + (a^2 - a^2e^2)m^2}{m^2+1} = \frac{b^2 + b^2m^2}{m^2+1}$$

$$= \boxed{b^2} \quad \text{Ans}$$