Q3 Prove that the locus of a point whose product of distances from the line $x+y=0$ & $x-y=0$ is constant is a hyperbola (s).

Soln

Let the point be (h, k)

$\perp^r$ dist from $x-y=0$ is $\left|\frac{h-k}{\sqrt{2}}\right|$

$\perp^r$ dist from $x+y=0$ is $\left|\frac{h+k}{\sqrt{2}}\right|$

Product is

$$\left|\frac{(h-k)(h+k)}{\sqrt{2}\cdot\sqrt{2}}\right| = \lambda^2 \text{ (const)}$$

$$h^2 - k^2 = \pm 2\lambda^2$$

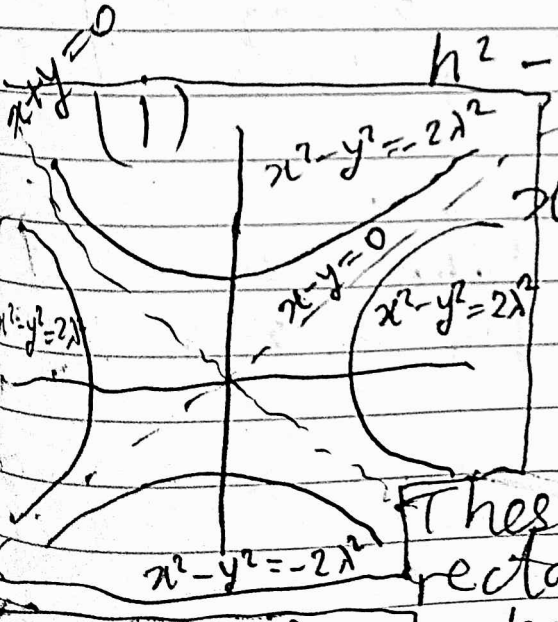
or

$$x^2 - y^2 = \pm(\sqrt{2}\lambda)^2$$

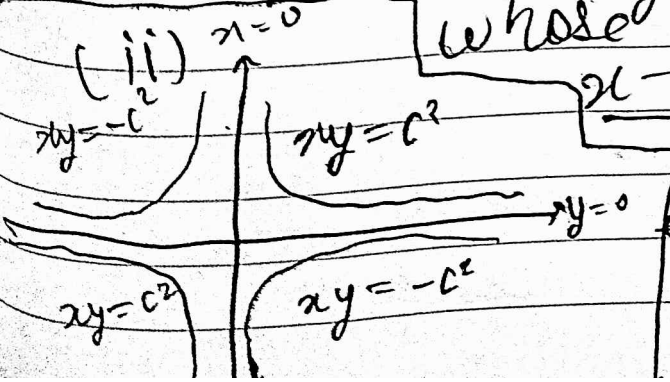
or

$$\boxed{\frac{x^2}{2\lambda^2} - \frac{y^2}{2\lambda^2} = \pm 1}$$

or



These are a set of rectangular hyperbolas whose assymptotes are $x-y=0$ and $x+y=0$



← rotate this (ii) system by 45^0 to get (i)