

1. If  $p$  and  $q$  are the lengths of the perpendiculars from the origin on the lines,  $x \operatorname{cosec} \alpha - y \sec \alpha = k \cot 2\alpha$  and  $x \sin \alpha + y \cos \alpha = k \sin 2\alpha$  respectively, then  $k^2$  is equal to :

**[Main Aug. 31, 2021 (I)]**

1. (a) Let  $L_1 = x \operatorname{cosec} \alpha - y \sec \alpha = k \cot 2\alpha$ .

$$\text{Then } \frac{x}{\sin \alpha} - \frac{y}{\cos \alpha} = \frac{k \cos 2\alpha}{\sin 2\alpha}$$

$$\Rightarrow x \cos \alpha - y \sin \alpha = \frac{k}{2} \cos 2\alpha$$

Perpendicular distance from (0 0) is

$$p = \left| \frac{0 - 0 - \frac{k}{2} \cos 2\alpha}{\sqrt{\cos^2 \alpha + \sin^2 \alpha}} \right|$$

$$\Rightarrow p = \left| \frac{k}{2} \cos 2\alpha \right| \Rightarrow 2p = |k \cos 2\alpha| \quad \dots(i)$$

$$L_2 = x \sin \alpha + y \cos \alpha = k \sin 2\alpha.$$

Perpendicular distance from (0, 0) is

$$q = \left| \frac{0 + 0 - k \sin 2\alpha}{\sqrt{\sin^2 \alpha + \cos^2 \alpha}} \right|$$

$$\Rightarrow q = |k \sin 2\alpha| \quad \dots(\text{ii})$$

Hence  $4p^2 + q^2 = k^3$  (from (i) & (ii))