Q. Supposing Newton's law of gravitation for gravitation forces $\mathbf{F}_1$ and $\mathbf{F}_2$ between two masses m_1 and m_2 at positions $\mathbf{r}_1$ and $\mathbf{r}_2$ read

$$\mathbf{F}_1 = -\mathbf{F}_2 = -\frac{\mathbf{r}_{12}}{r_{12}^3} GM^2{}_0 \left(\frac{m_1 m_2}{M_0^2}\right)^n$$

where M_0 is a constant of dimension of mass, $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ and n is a number. In such a case,

(a) the acceleration due to gravity on the earth will be different for different objects
(b) none of the three laws of Kepler will be valid
(c) only the third law will become invalid
(d) for n negative, an object lighter than water will sink in water

Ans. *(a,c, d)*

Given,
$$\mathbf{F}_1 = -\mathbf{F}_2 = \frac{-\mathbf{r}_{12}}{r_{12}^3} GM_0^2 \left(\frac{m_1 m_2}{M_0^2}\right)^n$$

$$\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$$

Acceleration due to gravity, $g = \dfrac{|F|}{\text{mass}}$

$$= \frac{GM_0^2 (m_1 m_2)^n}{r_{12}^2 (M_0)^{2n}} \times \frac{1}{(\text{mass})}$$

Since, g depends upon position vector, hence it will be different for different objects. As g is not constant, hence constant of proportionality will not be constant in Kepler's third law. Hence, Kepler's third law will not be valid.

As the force is of central nature. $\left[\because \text{force} \propto \dfrac{1}{r^2}\right]$

Hence, first two Kepler's laws will be valid.

For negative n,
$$g = \frac{GM_0^2 (m_1 m_2)^{-n}}{r_{12}^2 (M_0)^{-2n}} \times \frac{1}{(\text{mass})}$$

$$= \frac{GM_0^{2(1+n)}}{r_{12}^2} \frac{(m_1 m_2)^{-n}}{(\text{mass})}$$

$$g = \frac{GM_0^2}{r_{12}^2} \left(\frac{M_0^2}{m_1 m_2}\right)^n \times \frac{1}{\text{mass}}$$

As $M_0 > m_1$ or m_2

$g > 0$, hence in this case situation will reverse *i.e.*, object lighter than water will sink in water.