Q. If the law of gravitation, instead of being inverse square law, becomes an inverse cube law

- (a) planets will not have elliptic orbits
- (b) circular orbits of planets is not possible
- (c) projectile motion of a stone thrown by hand on the surface of the earth will be approximately parabolic

(d) there will be no gravitational force inside a spherical shell of uniform density

Ans. (a, c)

If the law of gravitation becomes an inverse cube law, then we can write, for a planet of mass m revolving around the sun of mass M,

$$F = \frac{GMm}{a^3} = \frac{mv^2}{a} \qquad \text{(where a is radius of orbiting planet)}$$

$$\Rightarrow \qquad v = \text{orbital speed} = \frac{\sqrt{GM}}{a} \Rightarrow v \propto \frac{1}{a}$$

Time period of revolution of a planet $T = \frac{2\pi a}{v} = \frac{2\pi a}{\frac{\sqrt{GM}}{a}} = \frac{2\pi a^2}{\sqrt{GM}}$

$$\Rightarrow \qquad T^2 \propto a^4$$

Hence, orbit will not be elliptical.

[for elliptical orbit $T^2 \propto a^3$]

As force

$$F = \left(\frac{GM}{a^3}\right)m = g'm$$
$$g' = \frac{GM}{a^3}$$

where,

As g', acceleration due to gravity is constant, hence path followed by a projectile will be approximately parabolic. (as $T \propto a^2$)