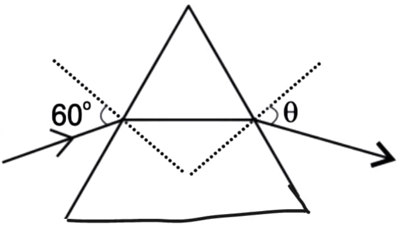
Question

A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle θ(n) with the normal. For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is



Solution

Correct option is A)

From the geometry we have $r_1 + r_2 = 60^\circ$

Using Snells law at the left side we have

$$\frac{\sin 60^\circ}{\sin r_1} = n$$

or

$$\sin r_1 = \frac{\sqrt{3}}{2n}$$

Using Snells law at the right end

$$\frac{\sin r_2}{\sin\theta} = \frac{1}{n}$$

or

$$\sin\theta = n\sin r_2$$

$$= n x in(60^\circ - r_1)$$

$$= n\left(\frac{\sqrt{3}}{2}\cos r_1 - \frac{1}{2}\sin r_1\right)$$

or

$$= n\left(\frac{\sqrt{3}}{2}\sqrt{1-\sin^2 r_1} - \frac{1}{2}\sin r_1\right)$$

or

$$= n\left(\frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4n^2}} - \frac{1}{2}\frac{\sqrt{3}}{2n}\right)$$

Solving we get

$$\sin\theta = \frac{\sqrt{3}}{4}\sqrt{4n^2-3} - \frac{\sqrt{3}}{4}$$

Differentiating

$$\cos\theta d\theta = \frac{\sqrt{3}}{4} \times \frac{1}{2\sqrt{4n^2-3}} \times 8n dn$$

or

$$\frac{d\theta}{dn} = \frac{\sqrt{3}n}{\sqrt{4n^2-3}} \times \frac{1}{\cos\theta}$$

$$= \frac{\sqrt{3}\times\sqrt{3}}{\sqrt{4\times3-3}} \times \frac{1}{1/2}$$

$$= 2$$