

Let f be a real valued differentiable function on \mathbb{R} (the set of all real numbers) such that $f(1) = 1$. If the y -intercept of the tangent at any point $P(x, y)$ on the curve $y = f(x)$ is equal to cube of abscissa of P , then the value of $f(-3)$ is equal to:

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Solution:

The tangent at point $P(x, y)$

$$\Rightarrow Y - y = \left(\frac{dy}{dx}\right)(X - x)$$

the y -intercept $\Rightarrow Y = y - x \frac{dy}{dx}$

\therefore It is given that y -intercept $= x^3$

$$\Rightarrow y - x \frac{dy}{dx} = x^3$$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = -x^2 \rightarrow \text{linear DE}$$

$$\text{IF} = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$$

\therefore the solution \Rightarrow

$$y \cdot \frac{1}{x} = \int -x^2 \cdot \frac{1}{x} dx + c$$

$$\Rightarrow \frac{y}{x} = -\frac{x^2}{2} + c$$

$\therefore f(1) = 1$ (given)

$$\Rightarrow 1 = -\frac{1}{2} + c \Rightarrow \boxed{c = \frac{3}{2}}$$

$$\Rightarrow \frac{y}{x} = -\frac{x^2}{2} + \frac{3}{2}$$

$$\boxed{\therefore y = \frac{-x^3 + 3x}{2}}$$

$$\therefore f(-3) = \frac{27 - 9}{2} = 9$$

$$\boxed{\therefore f(-3) = 9}$$