Q. Let C be a variable circle such that it touches the circle $x^2 + y^2 = 1$ and passes through the point $(4,3)$. Let r and (h,k) denote the radius and centre of C respectively, then which of the following statement(s) is(are) **TRUE**?

[A] $h^2 + k^2 = r^2 + 2r + 1$

[B] $r^2 = (h-4)^2 + (k-3)^2$

[C] The minimum value of r is 2

[D] The value r is minimum if $(h,k) = \left(\frac{12}{5}, \frac{9}{5}\right)$

Answer: [A][B][C][D]

Solution:

Condition of external touching $\Rightarrow d_{o_1o_2} = r_1 + r_2$

Centres: $(0,0)$ and (h,k)

$$\sqrt{h^2 + k^2} = r + 1 \quad \text{or} \quad h^2 + k^2 = (r+1)^2$$

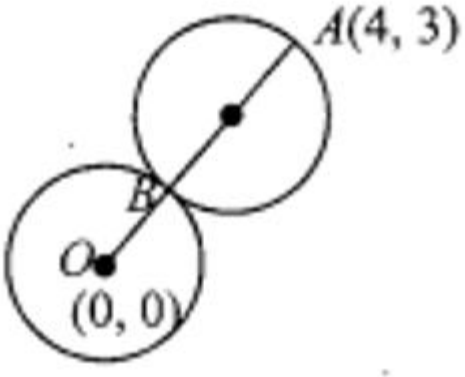
$$h^2 + k^2 = r^2 + 2r + 1$$

Hence Option [A] is correct

Since (3,4) lies on the circle, its distance from centre is r

$$(h-4)^2 + (k-3)^2 = r^2$$

Hence Option [B] is correct



The minimum value of r can be easily seen through analytical diagram.

The centre of both circles and thr point (4,3) must be collinear for diameter to be of minimum length.

So in minimum case, AB+OB=OA or $2r + 1 = 5$ or $r = 2$

And (h, k) =$\left(\frac{12}{5}, \frac{9}{5}\right)$ {section formula used}

Hence Option [C] and [D] are correct