Q. Let T_{ij} denote the total number of common tangents to circles C_i and C_j where $i, j \in \{1,2,3,4\}$. Given

$$C_1 : x^2 + y^2 - 2x - 6y + 9 = 0$$

$$C_2 : x^2 + y^2 - 2y + 6x + 1 = 0$$

$$C_3 : x^2 + y^2 - 2x - 3 = 0$$

$$C_4 : x^2 + y^2 - 2x - 2y + 1 = 0$$

Which of the following statement(s) is(are) **TRUE**?

[A] $T_{12} = 3$

[B] $T_{13} = T_{14} = 3$

[C] $T_{23} = 2$

[D] $T_{34} = 1$

Answer: [B][C][D]

Solution:

C_1: centre (1,3) & radius 1

C_2: centre $(-3,1)$ & radius 3

C_3: centre (1,0) & radius 2

C_4: centre (1,1) & radius 1

[A] $C_1C_2 = 2\sqrt{5}$ and $r_1 + r_2 = 1 + 3 = 4 \implies C_1C_2 > r_1 + r_2$; Thus we have 4 common tangents.

[B] $C_1C_3 = 3$ and $r_1 + r_3 = 1 + 2 = 3 \implies C_1C_3 = r_1 + r_3$; Thus we have 3 common tangents.

$C_1C_4 = 2$ and $r_1 + r_4 = 1 + 1 = 2 \implies C_1C_4 = r_1 + r_4$; Thus we have 3 common tangents.

(externally touching circles)

[C] $C_2C_3 = \sqrt{17}$ and $r_3 + r_2 = 2 + 3 = 5;\ |r_2 - r_3| = 1 \implies |r_2 - r_3| < C_3C_2 < r_3 + r_2$; Thus we have 2 common tangents.

[D] $C_3C_4 = 1$ and $|r_4 - r_3| = 2 - 1 = 1 \implies C_3C_4 = |r_4 - r_3|$; Thus we have only 1 common tangent. (C_3 and C_4 are internally touching)