

then, the displacement produced by  $S_2$  would be

$$y_2 = a \cos (\omega t + \phi)$$

and the resultant displacement will be given by

$$\begin{aligned} y &= y_1 + y_2 \\ &= a [\cos \omega t + \cos (\omega t + \phi)] \\ &= 2 a \cos (\phi / 2) \cos (\omega t + \phi / 2) \\ &\left[ \because \cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \right] \end{aligned}$$

The amplitude of the resultant displacement is  $2 a \cos (\phi / 2)$  and therefore the intensity at that point will be

$$I = 4 I_0 \cos^2 (\phi / 2) \tag{10.12}$$

If  $\phi = 0, \pm 2 \pi, \pm 4 \pi, \dots$  which corresponds to the condition given by Eq. (10.10) we will have constructive interference leading to maximum intensity. On the other hand, if  $\phi = \pm \pi, \pm 3\pi, \pm 5\pi \dots$  [which corresponds to the condition given by Eq. (10.11)] we will have destructive interference leading to zero intensity.

Now if the two sources are coherent (i.e., if the two needles are going up and down regularly) then the phase difference  $\phi$  at any point will not change with time and we will have a stable interference pattern; i.e., the positions of maxima and minima will not change with time. However, if the two needles do not maintain a constant phase difference, then the interference pattern will also change with time and, if the phase difference changes very rapidly with time, the positions of maxima and minima will also vary rapidly with time and we will see a “time-averaged” intensity distribution. When this happens, we will observe an average intensity that will be given by

$$\langle I \rangle = 4 I_0 \langle \cos^2 (\phi / 2) \rangle \tag{10.13}$$

where angular brackets represent time averaging. Indeed it is shown in Section 7.2 that if  $\phi(t)$  varies randomly with time, the time-averaged quantity  $\langle \cos^2 (\phi / 2) \rangle$  will be  $1/2$ . This is also intuitively obvious because the function  $\cos^2 (\phi / 2)$  will randomly vary between 0 and 1 and the average value will be  $1/2$ . The resultant intensity will be given by

$$I = 2 I_0 \tag{10.14}$$

at all points.

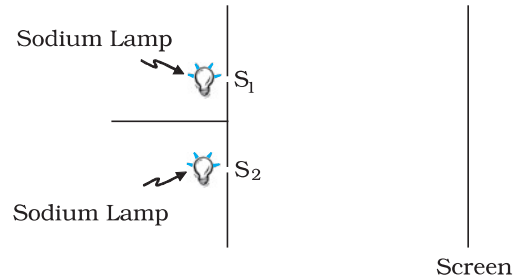
When the phase difference between the two vibrating sources changes rapidly with time, we say that the two sources are incoherent and when this happens the intensities just add up. This is indeed what happens when two separate light sources illuminate a wall.

## 10.5 INTERFERENCE OF LIGHT WAVES AND YOUNG’S EXPERIMENT

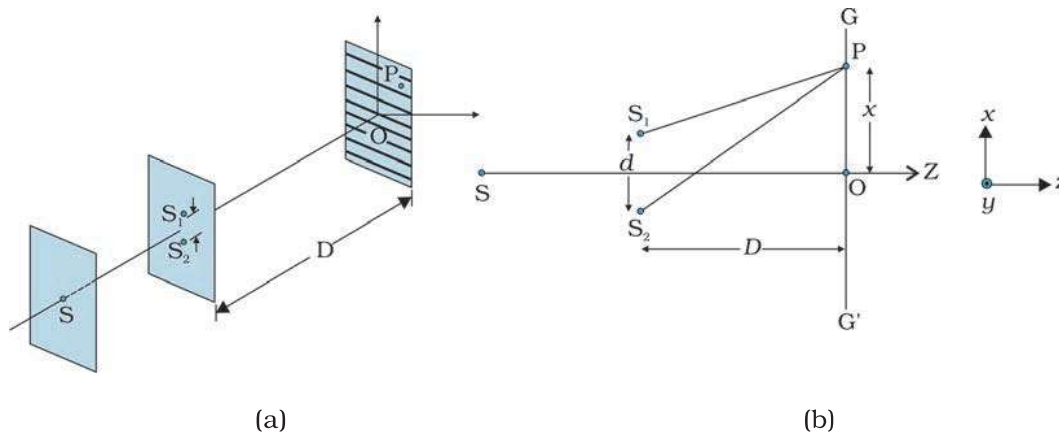
We will now discuss interference using light waves. If we use two sodium lamps illuminating two pinholes (Fig. 10.11) we will not observe any interference fringes. This is because of the fact that the light wave emitted from an ordinary source (like a sodium lamp) undergoes abrupt phase

changes in times of the order of  $10^{-10}$  seconds. Thus the light waves coming out from two independent sources of light will not have any fixed phase relationship and would be incoherent, when this happens, as discussed in the previous section, the intensities on the screen will add up.

The British physicist Thomas Young used an ingenious technique to “lock” the phases of the waves emanating from  $S_1$  and  $S_2$ . He made two pinholes  $S_1$  and  $S_2$  (very close to each other) on an opaque screen [Fig. 10.12(a)]. These were illuminated by another pinhole that was in turn, lit by a bright source. Light waves spread out from  $S$  and fall on both  $S_1$  and  $S_2$ .  $S_1$  and  $S_2$  then behave like two coherent sources because light waves coming out from  $S_1$  and  $S_2$  are derived from the same original source and any abrupt phase change in  $S$  will manifest in exactly similar phase changes in the light coming out from  $S_1$  and  $S_2$ . Thus, the two sources  $S_1$  and  $S_2$  will be *locked* in phase; i.e., they will be coherent like the two vibrating needle in our water wave example [Fig. 10.8(a)].



**FIGURE 10.11** If two sodium lamps illuminate two pinholes  $S_1$  and  $S_2$ , the intensities will add up and no interference fringes will be observed on the screen.



**FIGURE 10.12** Young’s arrangement to produce interference pattern.

Thus spherical waves emanating from  $S_1$  and  $S_2$  will produce interference fringes on the screen  $GG'$ , as shown in Fig. 10.12(b). The positions of maximum and minimum intensities can be calculated by using the analysis given in Section 10.4 where we had shown that for an arbitrary point  $P$  on the line  $GG'$  [Fig. 10.12(b)] to correspond to a maximum, we must have

$$S_2P - S_1P = n\lambda; \quad n = 0, 1, 2 \dots \quad (10.15)$$

Now,

$$(S_2P)^2 - (S_1P)^2 = \left[ D^2 + \left( x + \frac{d}{2} \right)^2 \right] - \left[ D^2 + \left( x - \frac{d}{2} \right)^2 \right] = 2xd$$



**Thomas Young (1773 – 1829)** English physicist, physician and Egyptologist. Young worked on a wide variety of scientific problems, ranging from the structure of the eye and the mechanism of vision to the decipherment of the Rosetta stone. He revived the wave theory of light and recognised that interference phenomena provide proof of the wave properties of light.

where  $S_1S_2 = d$  and  $OP = x$ . Thus

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P} \quad (10.16)$$

If  $x, d \ll D$  then negligible error will be introduced if  $S_2P + S_1P$  (in the denominator) is replaced by  $2D$ . For example, for  $d = 0.1$  cm,  $D = 100$  cm,  $OP = 1$  cm (which correspond to typical values for an interference experiment using light waves), we have

$$S_2P + S_1P = [(100)^2 + (1.05)^2]^{1/2} + [(100)^2 + (0.95)^2]^{1/2} \approx 200.01 \text{ cm}$$

Thus if we replace  $S_2P + S_1P$  by  $2D$ , the error involved is about 0.005%. In this approximation, Eq. (10.16) becomes

$$S_2P - S_1P \approx \frac{xd}{D} \quad (10.17)$$

Hence we will have constructive interference resulting in a bright region when  $\frac{xd}{D} = n\lambda$  [Eq. (10.15)]. That is,

$$x = x_n = \frac{n\lambda D}{d}; n = 0, \pm 1, \pm 2, \dots \quad (10.18)$$

On the other hand, we will have destructive interference resulting in a dark region when  $\frac{xd}{D} = (n + \frac{1}{2})\lambda$

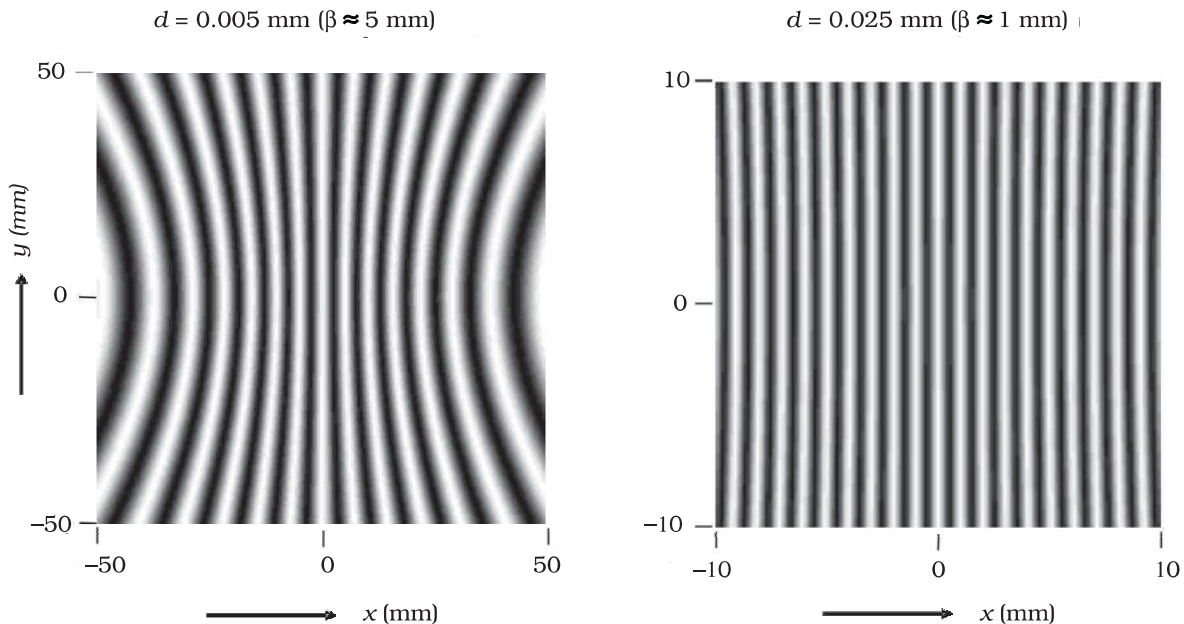
that is

$$x = x_n = (n + \frac{1}{2}) \frac{\lambda D}{d}; n = 0, \pm 1, \pm 2 \quad (10.19)$$

Thus dark and bright bands appear on the screen, as shown in Fig. 10.13. Such bands are called *fringes*. Equations (10.18) and (10.19) show that dark and bright fringes are equally spaced and the distance between two consecutive bright and dark fringes is given by

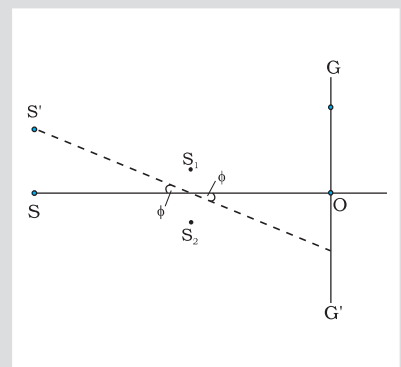
$$\beta = x_{n+1} - x_n \text{ or } \beta = \frac{\lambda D}{d} \quad (10.20)$$

which is the expression for the *fringe width*. Obviously, the central point O (in Fig. 10.12) will be bright because  $S_1O = S_2O$  and it will correspond to  $n = 0$  [Eq. (10.18)]. If we consider the line perpendicular to the plane of the paper and passing through O [i.e., along the  $y$ -axis] then all points on this line will be equidistant from  $S_1$  and  $S_2$  and we will have a bright central fringe which is a straight line as shown in Fig. 10.13. In order to determine the shape of the interference pattern on the screen we note that a particular fringe would correspond to the locus of points with a constant value of  $S_2P - S_1P$ . Whenever this constant is an integral multiple of  $\lambda$ , the fringe will be bright and whenever it is an odd integral multiple of  $\lambda/2$  it will be a dark fringe. Now, the locus of the point P lying in the  $x$ - $y$  plane such that  $S_2P - S_1P (= \Delta)$  is a constant, is a hyperbola. Thus the fringe pattern will strictly be a hyperbola; however, if the distance  $D$  is very large compared to the fringe width, the fringes will be very nearly straight lines as shown in Fig. 10.13.



**FIGURE 10.13** Computer generated fringe pattern produced by two point source  $S_1$  and  $S_2$  on the screen  $GG'$  (Fig. 10.12); (a) and (b) correspond to  $d = 0.005$  mm and  $0.025$  mm, respectively (both figures correspond to  $D = 5$  cm and  $\lambda = 5 \times 10^{-5}$  cm.) (Adopted from OPTICS by A. Ghatak, Tata McGraw Hill Publishing Co. Ltd., New Delhi, 2000.)

In the double-slit experiment shown in Fig. 10.12(b), we have taken the source hole  $S$  on the perpendicular bisector of the two slits, which is shown as the line  $SO$ . What happens if the source  $S$  is slightly away from the perpendicular bisector. Consider that the source is moved to some new point  $S'$  and suppose that  $Q$  is the mid-point of  $S_1$  and  $S_2$ . If the angle  $S'QS$  is  $\phi$ , then the central bright fringe occurs at an angle  $-\phi$ , on the other side. Thus, if the source  $S$  is on the perpendicular bisector, then the central fringe occurs at  $O$ , also on the perpendicular bisector. If  $S$  is shifted by an angle  $\phi$  to point  $S'$ , then the central fringe appears at a point  $O'$  at an angle  $-\phi$ , which means that it is shifted by the same angle on the other side of the bisector. This also means that the source  $S'$ , the mid-point  $Q$  and the point  $O'$  of the central fringe are in a straight line.



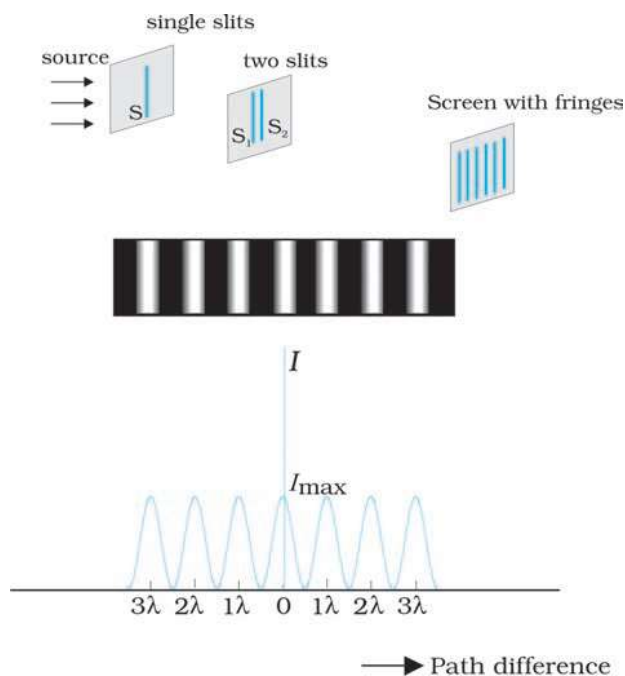
We end this section by quoting from the Nobel lecture of Dennis Gabor\*

*The wave nature of light was demonstrated convincingly for the first time in 1801 by Thomas Young by a wonderfully simple experiment. He let a ray of sunlight into a dark room, placed a dark screen in front of it, pierced with two small pinholes, and beyond this, at some distance, a white screen. He then saw two darkish lines at both sides of a bright line, which gave him sufficient encouragement to repeat the experiment, this time with spirit flame as light source, with a little salt in it to produce the bright yellow sodium light. This time he saw a number of dark lines, regularly spaced; the first clear proof that light added to light can produce darkness. This phenomenon is called*

\* Dennis Gabor received the 1971 Nobel Prize in Physics for discovering the principles of holography.

interference. Thomas Young had expected it because he believed in the wave theory of light.

We should mention here that the fringes are straight lines although  $S_1$  and  $S_2$  are point sources. If we had slits instead of the point sources (Fig. 10.14), each pair of points would have produced straight line fringes resulting in straight line fringes with increased intensities.



**FIGURE 10.14** Photograph and the graph of the intensity distribution in Young's double-slit experiment.

Interactive animation of Young's experiment  
<http://vsg.quasihome.com/interfer.html>



**EXAMPLE 10.3**

**Example 10.3** Two slits are made one millimetre apart and the screen is placed one metre away. What is the fringe separation when blue-green light of wavelength 500 nm is used?

**Solution** Fringe spacing  $= \frac{D\lambda}{d} = \frac{1 \times 5 \times 10^{-7}}{1 \times 10^{-3}} \text{ m}$   
 $= 5 \times 10^{-4} \text{ m} = 0.5 \text{ mm}$

**EXAMPLE 10.4**

**Example 10.4** What is the effect on the interference fringes in a Young's double-slit experiment due to each of the following operations:

- (a) the screen is moved away from the plane of the slits;
- (b) the (monochromatic) source is replaced by another (monochromatic) source of shorter wavelength;
- (c) the separation between the two slits is increased;
- (d) the source slit is moved closer to the double-slit plane;
- (e) the width of the source slit is increased;
- (f) the monochromatic source is replaced by a source of white light?

(In each operation, take all parameters, other than the one specified, to remain unchanged.)

**Solution**

- (a) Angular separation of the fringes remains constant ( $= \lambda/d$ ). The actual separation of the fringes increases in proportion to the distance of the screen from the plane of the two slits.
- (b) The separation of the fringes (and also angular separation) decreases. See, however, the condition mentioned in (d) below.
- (c) The separation of the fringes (and also angular separation) decreases. See, however, the condition mentioned in (d) below.
- (d) Let  $s$  be the size of the source and  $S$  its distance from the plane of the two slits. For interference fringes to be seen, the condition  $s/S < \lambda/d$  should be satisfied; otherwise, interference patterns produced by different parts of the source overlap and no fringes are seen. Thus, as  $S$  decreases (i.e., the source slit is brought closer), the interference pattern gets less and less sharp, and when the source is brought too close for this condition to be valid, the fringes disappear. Till this happens, the fringe separation remains fixed.
- (e) Same as in (d). As the source slit width increases, fringe pattern gets less and less sharp. When the source slit is so wide that the condition  $s/S \leq \lambda/d$  is not satisfied, the interference pattern disappears.
- (f) The interference patterns due to different component colours of white light overlap (incoherently). The central bright fringes for different colours are at the same position. Therefore, the central fringe is white. For a point  $P$  for which  $S_2P - S_1P = \lambda_b/2$ , where  $\lambda_b$  ( $\approx 4000 \text{ \AA}$ ) represents the wavelength for the blue colour, the blue component will be absent and the fringe will appear red in colour. Slightly farther away where  $S_2Q - S_1Q = \lambda_r/2$  where  $\lambda_r$  ( $\approx 8000 \text{ \AA}$ ) is the wavelength for the red colour, the fringe will be predominantly blue.

Thus, the fringe closest on either side of the central white fringe is red and the farthest will appear blue. After a few fringes, no clear fringe pattern is seen.

EXAMPLE 10.4

## 10.6 DIFFRACTION

If we look clearly at the shadow cast by an opaque object, close to the region of geometrical shadow, there are alternate dark and bright regions just like in interference. This happens due to the phenomenon of diffraction. Diffraction is a general characteristic exhibited by all types of waves, be it sound waves, light waves, water waves or matter waves. Since the wavelength of light is much smaller than the dimensions of most obstacles; we do not encounter diffraction effects of light in everyday observations. However, the finite resolution of our eye or of optical