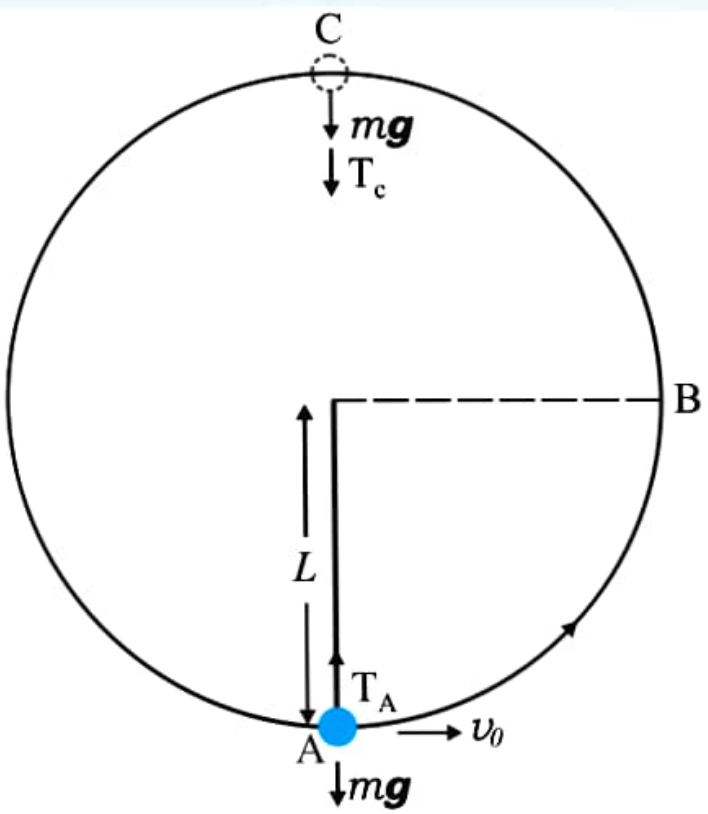
Example A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity v_o at the lowest point A such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, C. This is shown in Fig. 6.6. Obtain an expression for (i) v_o; (ii) the speeds at points B and C; (iii) the ratio of the kinetic energies (K_B/K_C) at B and C. Comment on the nature of the trajectory of the bob after it reaches the point C.



Answer (i) There are two external forces on the bob : gravity and the tension (T) in the string. The latter does no work since the displacement of the bob is always normal to the string. The potential energy of the bob is thus associated with the gravitational force only. The total mechanical energy E of the system is conserved. We take the potential energy of the system to be zero at the lowest point A. Thus, at A :

$$E = \frac{1}{2} m v_0^2 \qquad (6.12)$$

$$T_A - mg = \frac{m v_0^2}{L} \text{ [Newton's Second Law]}$$

where T_A is the tension in the string at A. At the highest point C, the string slackens, as the tension in the string (T_C) becomes zero.

Thus, at C

$$E = \frac{1}{2} m v_c^2 + 2mgL \qquad (6.13)$$

$$mg = \frac{m v_c^2}{L} \quad \text{[Newton's Second Law]} \qquad (6.14)$$

where v_C is the speed at C. From Eqs. (6.13) and (6.14)

$$E = \frac{5}{2} mgL$$

Equating this to the energy at A

$$\frac{5}{2} mgL = \frac{m}{2} v_0^2$$

or, $v_0 = \sqrt{5gL}$

(ii) It is clear from Eq. (6.14)

$$v_C = \sqrt{gL}$$

At B, the energy is

$$E = \frac{1}{2} m v_B^2 + mgL$$

Equating this to the energy at A and employing the result from (i), namely $v_0^2 = 5gL$,

$$\frac{1}{2} m v_B^2 + mgL = \frac{1}{2} m v_0^2$$

$$= \frac{5}{2} m g L$$

$$\therefore v_B = \sqrt{3gL}$$

(iii) The ratio of the kinetic energies at B and C is :

$$\frac{K_B}{K_C} = \frac{\frac{1}{2}mv_B^2}{\frac{1}{2}mv_C^2} = \frac{3}{1}$$

At point C, the string becomes slack and the velocity of the bob is horizontal and to the left. If the connecting string is cut at this instant, the bob will execute a projectile motion with horizontal projection akin to a rock kicked horizontally from the edge of a cliff. Otherwise the bob will continue on its circular path and complete the revolution.