Find the general solution of the DE:

$$\frac{dx}{dy} = \frac{y\tan y - x\tan y - xy}{y\tan y}$$

Solution:

Given, $$\frac{dx}{dy} = \frac{y\tan y - x\tan y - xy}{y\tan y}$$

$$\Rightarrow \frac{dx}{dy} + \left(\frac{1}{y} + \frac{1}{\tan y}\right)x = 1$$

comparing with $\frac{dx}{dy} + Py = Q$, we have:

$P = \frac{1}{y} + \frac{1}{\tan y}$ and $Q = 1$

$$\therefore IF = e^{\int\left(\frac{1}{y} + \frac{1}{\tan y}\right)dy} = e^{(\log y + \log\sin y)}$$

$$= e^{\log(y\sin y)}$$

$$= y\sin y$$

$\therefore$ The solution is

$$\Rightarrow x \cdot IF = \int (Q \cdot IF)\,dy + c$$

$$\Rightarrow x(y\sin y) = \int y\sin y\,dy + c$$

$$\Rightarrow xy\sin y = \int y\sin y\,dy + c$$

integrating by parts,

$$\Rightarrow xy\sin y = y(-\cos y) + \int \cos y\,dy + c$$

$$\Rightarrow xy\sin y = -y\cos y + \sin y + c$$

$$\Rightarrow \boxed{x = \frac{\sin y - y\cos y + c}{y\sin y}}$$