

The general solution of the DE $y(x^2y + e^x) dx - e^x dy = 0$ is

Solution:

$$\text{Given, } y(x^2y + e^x) dx - e^x dy = 0$$

$$\Rightarrow x^2y^2 + ye^x dx = e^x dy$$

$$\Rightarrow \frac{1}{y^2} \frac{dy}{dx} - \frac{1}{y} = \frac{x^2}{e^x}$$

$$\text{let } \frac{1}{y} = t$$

$$\Rightarrow \frac{-1}{y^2} \frac{dy}{dx} = \frac{dt}{dx}$$

$$\therefore \frac{dt}{dx} + t = -\frac{x^2}{e^x}$$

this is a linear DE

$$\text{IF} = e^{\int 1 dx} = e^x$$

\therefore the solution is

$$t \cdot e^x = \int \frac{-x^2}{e^x} \times e^x dx$$

$$t \cdot e^x = -\frac{x^3}{3} + c \Rightarrow \frac{1}{y} e^x = -\frac{x^3}{3} + c$$

$$\boxed{\therefore 3e^x + x^3y = 3cy}$$