

Solution of the DE: $2xy \cdot \frac{dy}{dx} = x^2 + 3y^2$ is

Ans:

$$\text{Given, } 2xy \cdot \frac{dy}{dx} = x^2 + 3y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{x^2 + 3v^2 x^2}{2vx^2}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1 + 3v^2}{2v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1 + v^2}{2v}$$

$$\Rightarrow \frac{2v dv}{1 + v^2} = \frac{dx}{x}$$

on integrating,

$$\log(1 + v^2) = \log(cx)$$

$$\Rightarrow 1 + v^2 = cx$$

$$\boxed{\therefore x^2 + y^2 = cx^3}$$