

A curve passes through the point $(1, \frac{\pi}{6})$. If the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right)$, $x > 0$, then its equation is:

Solution:

$$\text{Given: } \frac{dy}{dx} = \frac{y}{x} + \sec\left(\frac{y}{x}\right)$$

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\Rightarrow v + x \frac{dv}{dx} = v + \sec v$$

$$\Rightarrow dv \cdot \cos v = \frac{dx}{x}$$

integrating on both sides

$$\Rightarrow \sin v = \log x + c$$

\therefore the curve passes through $(1, \frac{\pi}{6})$

$$\Rightarrow c = \frac{1}{2}$$

$$\therefore \sin \frac{y}{x} = \log x + \frac{1}{2}$$