If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is:

Solution:

Given, $\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$

put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v(\log v + 1)$

$\Rightarrow x\frac{dv}{dx} = v\log v$

$\Rightarrow \frac{dv}{v\log v} = \frac{dx}{x}$

integrating on both sides

$\Rightarrow \log(\log v) = \log cx$

$\Rightarrow \log v = cx$

$\therefore \log\left(\frac{y}{x}\right) = cx$