

The solution of $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$

Solution:

$$\text{Given, } \frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$$

$$\text{put } y = vx \Rightarrow v + x \frac{dv}{dx} = \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = v + \tan v$$

$$\Rightarrow v + x \frac{dv}{dx} = v + \tan v$$

$$\Rightarrow x \frac{dv}{dx} = \tan v$$

$$\Rightarrow \cot v dv = \frac{dx}{x}$$

on integrating,

$$\ln |\sin v| = \ln x + \ln k$$

$$\Rightarrow \sin v = kx$$

$$\boxed{\therefore \sin \frac{y}{x} = kx}$$