

Equation of the curve through the point $(1,0)$ which satisfies the DE:

$$(1+y^2)dx - xy dy = 0 \text{ is } \underline{\hspace{2cm}}$$

Solution:

$$\text{Given, } (1+y^2)dx - xy dy = 0$$

$$\Rightarrow \frac{dx}{x} = \frac{y dy}{1+y^2}$$

integrating,

$$\Rightarrow \int \frac{dx}{x} = \int \frac{y dy}{1+y^2}$$

$$\Rightarrow \log|x| = \frac{1}{2} \log|1+y^2| + \log c$$

$$|x| = c\sqrt{1+y^2}$$

\therefore the curve passes through $(1,0)$

$$\Rightarrow 1 = c\sqrt{1+0}$$

$$\Rightarrow c = 1$$

$$\therefore |x| = \sqrt{1+y^2}$$

$$\Rightarrow x^2 = 1+y^2$$

$$\boxed{\therefore x^2 - y^2 = 1}$$