A function $y=f(x)$ has the second order derivatives $f''(x)=6(x-1)$. If its graph passes through the point $(2,1)$ and at that point the tangent to the graph is $y=3x-5$, then the function is ——

Solution:

Given $f''(x)=6(x-1)$

$\Rightarrow f'(x)=3(x-1)^2+c_1$

But at point $(2,1)$, the line $y=3x-5$ is tangent to the graph $f(x)=y$

$\therefore \left.\frac{dy}{dx}\right|_{x=2}=3$

or $f'(2)=3$.

$\Rightarrow 3=3(2-1)^2+c_1$

$\Rightarrow \boxed{c_1=0}$

$\therefore f'(x)=3(x-1)^2$

$\Rightarrow f(x)=(x-1)^3+c_2$

$\because$ It is given that $f(2)=1$

$\Rightarrow f(2)=(2-1)^3+c_2$

$\Rightarrow c_2=0$

$$\boxed{\therefore f(x)=(x-1)^3}$$