

Solve the differential equation:

$$(1-y^2)(1+\log x)dx + 2xydy = 0, \text{ given that}$$

$$\text{when } x=1, y=0$$

Solution:

$$(1-y^2)(1+\log x)dx + 2xydy = 0$$

$$\Rightarrow \frac{1+\log x}{x} dx = \frac{-2y}{1-y^2} dy$$

(Variable-seperable)

on integrating,

$$\int \frac{1+\log x}{x} dx = \int \frac{-2y}{1-y^2} dy$$

$$\Rightarrow \frac{(1+\log x)^2}{2} = \log |1-y^2| + c \rightarrow \textcircled{1}$$

$$\text{when } x=1, y=0$$

$$\Rightarrow \frac{(1+\log 1)^2}{2} = \log |1| + c$$

$$\Rightarrow \boxed{c = 1/2}$$

substituting c in $\textcircled{1}$

$$\Rightarrow \frac{(1+\log x)^2}{2} = \log |1-y^2| + \frac{1}{2}$$

$$\boxed{\therefore (1+\log x)^2 = 2 \log |1-y^2| + 1}$$