The curve which satisfies the DE, $\frac{x\,dy - y\,dx}{x\,dy + y\,dx} = y^2 \sin(xy)$ and passes through $(0,1)$ is given by:

Solution:

Given DE can be written as:

$$\left(\frac{x\,dy - y\,dx}{x^2}\right)\left(\frac{x^2}{y^2}\right) = (x\,dy + y\,dx)\sin(xy)$$

$$\Rightarrow \quad d\left(\frac{y}{x}\right)\left(\frac{x^2}{y^2}\right) = d(xy)\sin(xy)$$

integrating on both sides

$$\Rightarrow \quad \frac{-1}{y/x} = -\cos xy + c \quad \Rightarrow \quad \frac{x}{y} = \cos(xy) - c$$

$\because$ the curve passes through $(0,1)$

$$\Rightarrow \quad c = 1$$

$$\boxed{\therefore \quad \frac{x}{y} = \cos(xy) - 1}$$

Scanned with CamScanner