$(x^2+y^2)dy = xy\,dx$. If $y(x_0)=e$, $y(1)=1$, then the value of $x_0 =$

Solution:

Given, $(x^2+y^2)\,dy = xy\,dx$

$$\Rightarrow x^2 dy - xy\,dx = -y^2 dy$$

$$\Rightarrow x(x\,dy - y\,dx) = -y^2 dy$$

$$\Rightarrow \frac{x(y\,dx - x\,dy)}{y^2} = dy.$$

$$\Rightarrow \left(\frac{x}{y}\right)\left(\frac{y\,dx - x\,dy}{y^2}\right) = \frac{dy}{y}$$

$$\Rightarrow \left(\frac{x}{y}\right) d\left(\frac{x}{y}\right) = \frac{dy}{y}$$

integrating on both sides

$$\Rightarrow \frac{x^2}{2y^2} = \log_e y + c$$

$\because$ given that $y(1)=1 \Rightarrow c = 1/2$

$$\Rightarrow \frac{x^2}{2y^2} = \log_e y + \frac{1}{2}$$

now $y(x_0) = e$

$$\Rightarrow \frac{x_0^2}{2e^2} = \log_e e + \frac{1}{2}$$

$$\Rightarrow \frac{x_0^2}{\cancel{2}e^2} = \frac{3}{\cancel{2}} \Rightarrow x_0^2 = 3e^2$$

$$\boxed{\therefore x_0 = \sqrt{3}e}$$