5.10 CIRCULAR MOTION

We have seen in Chapter 4 that acceleration of a body moving in a circle of radius R with uniform speed v is v^2/R directed towards the centre. According to the second law, the force f_c providing this acceleration is :

$$f_c = \frac{mv^2}{R} \quad (5.16)$$

where m is the mass of the body. This force directed forwards the centre is called the centripetal force. For a stone rotated in a circle by a string, the centripetal force is provided by the tension in the string. The centripetal force for motion of a planet around the sun is the gravitational force on the planet due to the sun. For a car taking a circular turn on a horizontal road, the centripetal force is the force of friction.

The circular motion of a car on a flat and banked road give interesting application of the laws of motion.

Motion of a car on a level road

Three forces act on the car (Fig. 5.14(a):

(i) The weight of the car, mg

(ii) Normal reaction, N

(iii) Frictional force, f

As there is no acceleration in the vertical direction

$N - mg = 0$

$N = mg$ (5.17)

The centripetal force required for circular motion is along the surface of the road, and is provided by the component of the contact force between road and the car tyres along the surface. This by definition is the frictional force. Note that it is the static friction that provides the centripetal acceleration. Static friction opposes the impending motion of the car moving away from the circle. Using equation (5.14) & (5.16) we get the result

$$f = \frac{mv^2}{R} \le \mu_s N$$

$$v^2 \le \frac{\mu_s RN}{m} = \mu_s Rg \qquad [\because N = mg]$$

which is independent of the mass of the car. This shows that for a given value of μ_s and R, there is a maximum speed of circular motion of the car possible, namely

$$v_{max} = \sqrt{\mu_s Rg} \quad (5.18)$$

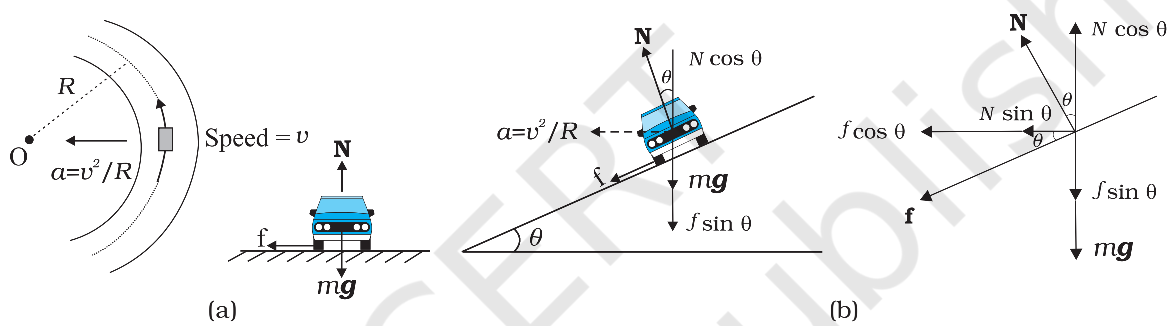

Fig. 5.14 Circular motion of a car on (a) a level road, (b) a banked road.

Motion of a car on a banked road

We can reduce the contribution of friction to the circular motion of the car if the road is banked (Fig. 5.14(b)). Since there is no acceleration along the vertical direction, the net force along this direction must be zero. Hence,

$$N\cos\theta = mg + f\sin\theta \quad (5.19a)$$

The centripetal force is provided by the horizontal components of N and f.

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R} \quad (5.19b)$$

But $f \le \mu_s N$

Thus to obtain v_{max} we put

$f = \mu_s N$.

Then Eqs. (5.19a) and (5.19b) become

$$N\cos\theta = mg + \mu_s N \sin\theta \quad (5.20a)$$

$N \sin\theta + \mu_s N \cos\theta = mv^2/R$ (5.20b)

From Eq. (5.20a), we obtain

$$N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

Substituting value of N in Eq. (5.20b), we get

$$\frac{mg(\sin\theta + \mu_s \cos\theta)}{\cos\theta - \mu_s \sin\theta} = \frac{mv_{max}^2}{R}$$

$$\text{or } v_{max} = \left(Rg\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)^{\frac{1}{2}} \quad (5.21)$$

Comparing this with Eq. (5.18) we see that maximum possible speed of a car on a banked road is greater than that on a flat road.

For $\mu_s = 0$ in Eq. (5.21),

$v_o = (R g \tan\theta)^{1/2}$ (5.22)

At this speed, frictional force is not needed at all to provide the necessary centripetal force. Driving at this speed on a banked road will cause little wear and tear of the tyres. The same equation also tells you that for $v < v_o$, frictional force will be up the slope and that a car can be parked only if $\tan\theta \le \mu_s$.

> **Example 5.10** A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

Answer On an unbanked road, frictional force alone can provide the centripetal force needed to keep the cyclist moving on a circular turn without slipping. If the speed is too large, or if the turn is too sharp (i.e. of too small a radius) or both, the frictional force is not sufficient to provide the necessary centripetal force, and the cyclist slips. The condition for the cyclist not to slip is given by Eq. (5.18) :

$$v^2 \le \mu_s R g$$

Now, $R = 3$ m, $g = 9.8$ m s^{-2}, $\mu_s = 0.1$. That is, $\mu_s R g = 2.94$ m^2 s^{-2}. $v = 18$ km/h $= 5$ m s^{-1}; i.e., $v^2 = 25$ m^2 s^{-2}. The condition is not obeyed. The cyclist will slip while taking the circular turn.

> **Example 5.11** A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the (a) optimum speed of the race-car to avoid wear and tear on its tyres, and (b) maximum permissible speed to avoid slipping ?

Answer On a banked road, the horizontal component of the normal force and the frictional force contribute to provide centripetal force to keep the car moving on a circular turn without slipping. At the optimum speed, the normal reaction's component is enough to provide the needed centripetal force, and the frictional force is not needed. The optimum speed v_o is given by Eq. (5.22):

$$v_o = (R g \tan\theta)^{1/2}$$

Here $R = 300$ m, $\theta = 15°$, $g = 9.8$ m s^{-2}; we have

$$v_o = 28.1 \text{ m s}^{-1}.$$

The maximum permissible speed v_{max} is given by Eq. (5.21):

$$v_{max} = \left(R g \frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)^{1/2} = 38.1 \text{ m s}^{-1}$$

5.11 SOLVING PROBLEMS IN MECHANICS

The three laws of motion that you have learnt in this chapter are the foundation of mechanics. You should now be able to handle a large variety of problems in mechanics. A typical problem in mechanics usually does not merely involve a single body under the action of given forces. More often, we will need to consider an assembly of different bodies exerting forces on each other. Besides, each body in the assembly experiences the force of gravity. When trying to solve a problem of this type, it is useful to remember the fact that we can choose any part of the assembly and apply the laws of motion to that part provided we include all forces on the chosen part due to the remaining parts of the assembly. We may call the chosen part of the assembly as the system and the remaining part of the assembly (plus any other agencies of forces) as the environment. We have followed the same method in solved examples. To handle a typical problem in mechanics systematically, one should use the following steps :

(i) Draw a diagram showing schematically the various parts of the assembly of bodies, the links, supports, etc.

(ii) Choose a convenient part of the assembly as one system.

(iii) Draw a separate diagram which shows this system and all the forces on the system by the remaining part of the assembly. Include also the forces on the system by other agencies. **Do not include the forces on the environment by the system**. A diagram of this type is known as 'a free-body diagram'. (Note this does not imply that the system under consideration is without a net force).

(iv) In a free-body diagram, include information about forces (their magnitudes and directions) that are either given or you are sure of (e.g., the direction of tension in a string along its length). The rest should be treated as unknowns to be determined using laws of motion.

(v) If necessary, follow the same procedure for another choice of the system. In doing so, employ Newton's third law. That is, if in the free-body diagram of *A*, the force on *A* due to *B* is shown as **F**, then in the free-body diagram of *B*, the force on *B* due to *A* should be shown as –**F**.

The following example illustrates the above procedure :

> ▶ ***Example 5.12*** See Fig. 5.15. A wooden block of mass 2 kg rests on a soft horizontal floor. When an iron cylinder of mass 25 kg is placed on top of the block, the floor yields steadily and the block and the cylinder together go down with an acceleration of 0.1 m s^{-2}. What is the action of the block on the floor (a) before and (b) after the floor yields ? Take g = 10 m s^{-2}. Identify the action-reaction pairs in the problem.

Answer

(a) The block is at rest on the floor. Its free-body diagram shows two forces on the block, the force of gravitational attraction by the earth equal to 2 × 10 = 20 N; and the normal force *R* of the floor on the block. By the First Law, the net force on the block must be zero i.e., *R* = 20 N. Using third law the action of the block (i.e. the force exerted on the floor by the block) is equal to 20 N and directed vertically downwards.

(b) The system (block + cylinder) accelerates downwards with 0.1 m s^{-2}. The free-body diagram of the system shows two forces on the system : the force of gravity due to the earth (270 N); and the normal force *R′* by the floor. Note, the free-body diagram of the system does not show the internal forces between the block and the cylinder. Applying the second law to the system,

$$270 - R' = 27 \times 0.1\text{N}$$
$$\text{ie. } R' = 267.3 \text{ N}$$

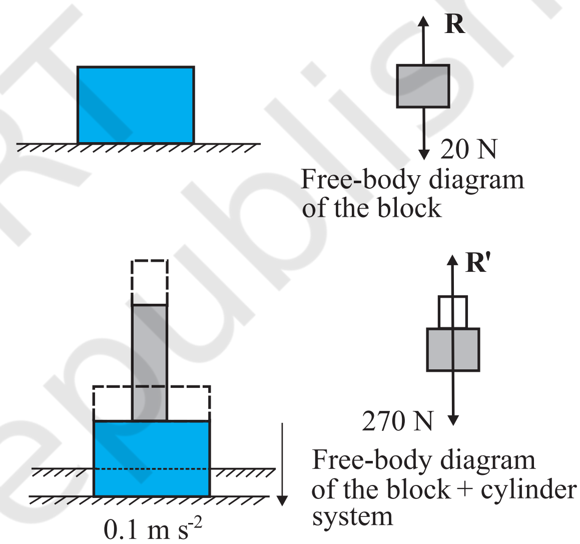

Fig. 5.15

By the third law, the action of the system on the floor is equal to 267.3 N vertically downward.

Action-reaction pairs

For (a): (i) the force of gravity (20 N) on the block by the earth (say, action); the force of gravity on the earth by the block (reaction) equal to 20 N directed upwards (not shown in the figure).
(ii) the force on the floor by the block (action); the force on the block by the floor (reaction).

For (b): (i) the force of gravity (270 N) on the system by the earth (say, action); the force of gravity on the earth by the system (reaction), equal to 270 N,

directed upwards (not shown in the figure).
(ii) the force on the floor by the system (action); the force on the system by the floor (reaction). In addition, for (b), the force on the block by the cylinder and the force on the cylinder by the block also constitute an action-reaction pair.

The important thing to remember is that an action-reaction pair consists of mutual forces which are always equal and opposite between two bodies. Two forces on the same body which happen to be equal and opposite can never constitute an action-reaction pair. The force of gravity on the mass in (a) or (b) and the normal force on the mass by the floor are not action-reaction pairs. These forces happen to be equal and opposite for (a) since the mass is at rest. They are not so for case (b), as seen already. The weight of the system is 270 N, while the normal force R' is 267.3 N. ◀

The practice of drawing free-body diagrams is of great help in solving problems in mechanics. It allows you to clearly define your system and consider all forces on the system due to objects that are not part of the system itself. A number of exercises in this and subsequent chapters will help you cultivate this practice.

SUMMARY

1. Aristotle's view that a force is necessary to keep a body in uniform motion is wrong. A force is necessary in practice to counter the opposing force of friction.
2. Galileo extrapolated simple observations on motion of bodies on inclined planes, and arrived at the law of inertia. Newton's first law of motion is the same law rephrased thus: *"Everybody continues to be in its state of rest or of uniform motion in a straight line, unless compelled by some external force to act otherwise".* In simple terms, the First Law is **"If external force on a body is zero, its acceleration is zero"**.
3. Momentum ($\mathbf{p}$) of a body is the product of its mass (m) and velocity ($\mathbf{v}$) :
   $$\mathbf{p} = m\mathbf{v}$$
4. Newton's second law of motion :
   The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts. Thus
   $$\mathbf{F} = k\frac{d\mathbf{p}}{dt} = k\,m\,\mathbf{a}$$
   where $\mathbf{F}$ is the net external force on the body and $\mathbf{a}$ its acceleration. We set the constant of proportionality k = 1 in SI units. Then
   $$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m\mathbf{a}$$
   The SI unit of force is newton : 1 N = 1 kg m s^{-2}.
   (a) The second law is consistent with the First Law ($\mathbf{F} = 0$ implies $\mathbf{a} = 0$)
   (b) It is a vector equation
   (c) It is applicable to a particle, and also to a body or a system of particles, provided $\mathbf{F}$ is the total external force on the system and $\mathbf{a}$ is the acceleration of the system as a whole.
   (d) $\mathbf{F}$ at a point at a certain instant determines $\mathbf{a}$ at the same point at that instant. That is the Second Law is a local law; $\mathbf{a}$ at an instant does not depend on the history of motion.
5. Impulse is the product of force and time which equals change in momentum.
   The notion of impulse is useful when a large force acts for a short time to produce a measurable change in momentum. Since the time of action of the force is very short, one can assume that there is no appreciable change in the position of the body during the action of the impulsive force.
6. Newton's third law of motion:
   To every action, there is always an equal and opposite reaction

In simple terms, the law can be stated thus :

Forces in nature always occur between pairs of bodies. Force on a body A by body B is equal and opposite to the force on the body B by A.

Action and reaction forces are simultaneous forces. There is no cause-effect relation between action and reaction. Any of the two mutual forces can be called action and the other reaction. Action and reaction act on different bodies and so they cannot be cancelled out. The internal action and reaction forces between different parts of a body do, however, sum to zero.

7. *Law of Conservation of Momentum*

   The total momentum of an isolated system of particles is conserved. The law follows from the second and third law of motion.

8. *Friction*

   Frictional force opposes (impending or actual) relative motion between two surfaces in contact. It is the component of the contact force along the common tangent to the surface in contact. Static friction f_s opposes impending relative motion; kinetic friction f_k opposes actual relative motion. They are independent of the area of contact and satisfy the following approximate laws :

$$f_s \le (f_s)_{max} = \mu_s R$$

$$f_k = \mu_k R$$

μ_s (co-efficient of static friction) and μ_k (co-efficient of kinetic friction) are constants characteristic of the pair of surfaces in contact. It is found experimentally that μ_k is less than μ_s .

Quantity	Symbol	Units	Dimensions	Remarks
Momentum	**p**	kg m s^{-1} or N s	[MLT^{-1}]	Vector
Force	**F**	N	[MLT^{-2}]	**F** = m **a** Second Law
Impulse		kg m s^{-1} or N s	[$M LT^{-1}$]	Impulse = force × time = change in momentum
Static friction	$\mathbf{f}_s$	N	[MLT^{-2}]	$\mathbf{f}_s \le \mu_s \mathbf{N}$
Kinetic friction	$\mathbf{f}_k$	N	[MLT^{-2}]	$\mathbf{f}_k = \mu_k \mathbf{N}$

POINTS TO PONDER

1. Force is not always in the direction of motion. Depending on the situation, **F** may be along **v**, opposite to **v**, normal to **v** or may make some other angle with **v**. In every case, it is parallel to acceleration.
2. If **v** = 0 at an instant, i.e. if a body is momentarily at rest, it does not mean that force or acceleration are necessarily zero at that instant. For example, when a ball thrown upward reaches its maximum height, **v** = 0 but the force continues to be its weight mg and the acceleration is not zero but g.
3. Force on a body at a given time is determined by the situation at the location of the body at that time. Force is not 'carried' by the body from its earlier history of motion. The moment after a stone is released out of an accelerated train, there is no horizontal force (or acceleration) on the stone, if the effects of the surrounding air are neglected. The stone then has only the vertical force of gravity.
4. In the second law of motion **F** = m **a**, **F** stands for the net force due to all material agencies external to the body. **a** is the effect of the force. m**a** should not be regarded as yet another force, besides **F**.