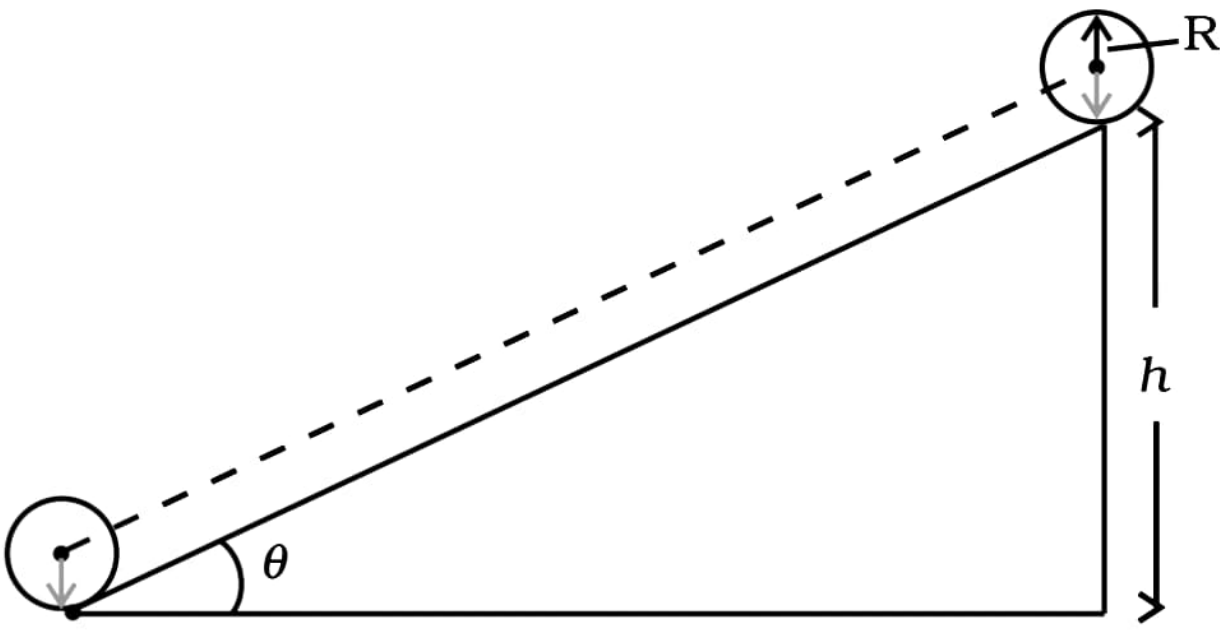
▶ ***Example 7.16*** Three bodies, a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of the bodies are identical. Which of the bodies reaches the ground with maximum velocity?

Answer We assume conservation of energy of the rolling body, i.e. there is no loss of energy due to friction etc. The potential energy lost by the body in rolling down the inclined plane (= *mgh*) must, therefore, be equal to kinetic energy gained. (See Fig) Since the bodies start from rest the kinetic energy gained is equal to the final kinetic energy of the bodies. From

$$K=\frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$$, where v is the final velocity of (the centre of mass of) the body. Equating K and mgh,



$$mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$$

$$\text{or } v^2 = \left(\frac{2gh}{1+k^2/R^2}\right)$$

Note is independent of the mass of the rolling body;

For a ring, $k^2 = R^2$

$$v_{ring} = \sqrt{\frac{2gh}{1+1}},$$

$$= \sqrt{gh}$$

For a solid cylinder $k^2 = R^2/2$

$$v_{disc} = \sqrt{\frac{2gh}{1+1/2}}$$

$$= \sqrt{\frac{4gh}{3}}$$

For a solid sphere $k^2 = 2R^2/5$

$$v_{sphere} = \sqrt{\frac{2gh}{1+2/5}}$$

$$= \sqrt{\frac{10gh}{7}}$$