

System of Linear Equations

Let A be an $n \times n$ matrix, x be a $n \times 1$ vector, and b be an $n \times 1$ vector

Then system of n linear equations in variable $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ can be

written as $Ax = b$ ——— ①

Any x which satisfies ① is said to be solution of system of the linear equations.

A system of linear equations can have

- i) Unique solution
- ii) infinitely many solutions
- iii) No solution.

Rank(A) can be obtained by reducing matrix A, using elementary row operations, to ^{Row} Echelon form

The number of non-zero rows of matrix in ^{Row} Echelon form gives the rank of a matrix.

1) If $\text{rank}(A) = \text{rank}(A|b) = n$

Then system of equations has unique solutions.

In this case $\det A \neq 0$

2) If $\text{rank}(A) = \text{rank}(A|b)$

$$= m < n$$

Then system has infinitely many solutions.

3) If $\text{rank}(A) \neq \text{rank}(A|b)$

Then the system of equations has no solution.