Q. A circle C touches the line $7x + 3y = 10$ at $(1,1)$ and passes through the point $(-1,7)$. If the point (X,Y) also lies on the circle, then the value of

$$\frac{(X - 1)^2 + (Y - 1)^2}{7X + 3Y - 10}$$

is _____

Answer: 10

Solution:

A family of circle touching a line at a point can be as the linear combination of of a point circle and that line $(S_0 + \lambda L = 0)$ie

$$C: \ (x-1)^2 + (y-1)^2 + \lambda(7x + 3y - 10) = 0$$

Now C passes through (-1,7); on putting we get

$$(-2)^2 + (6)^2 + \lambda(-7 + 21 - 10) = 0$$

$$\lambda = -10$$

Now C also passes through (X, Y); on putting we get

$$(X-1)^2 + (Y-1)^2 - 10(7X + 3Y - 10) = 0$$

$$\frac{(X-1)^2 + (Y-1)^2}{7X + 3Y - 10} = 10$$