Q. The centre and the radius of a circle that cuts the 3 circles $x^2 + y^2 - 3x - 6y + 14 = 0$, $x^2 + y^2 - x - 4y + 8 = 0$, $x^2 + y^2 + 2x - 6y + 9 = 0$ orthogonally is (h, k) and r respectively. The value of $2h + k - 2r$ is __________.

Answer: 0

Solution:

Method 1:

Simply apply condition of orthogonality 3 times ie

$$-3g - 6f = 14 + c$$

$$-g - 4f = 8 + c$$

$$2g - 6f = 9 + c$$

On solving these equations $g = -1,\ f = -2,\ c = 1$

Hence $(h, k) = (1,2)$ and $r = \sqrt{g^2 + f^2 - c} = 2$

$$2h + k - 2r = 0$$

Method 2:

Use property that the circle cutting 3 circles orthogonally has its centre at the orthocenter of 3 circles ie point of intersection of any 2 radical axes. And its radius is length of tangent from orthocenter to any of the 3 circles.

radical axes are $S_1 - S_2 = 0$ and $S_2 - S_3 = 0$ and $S_3 - S_1 = 0$

any two are $x + y - 3 = 0$ and $3x - 2y + 1 = 0 \implies$ on solving we get (1,2)

hence $(h, k) = (1,2)$

Length of tangent from (1,2) is $\sqrt{S_1(1,2)} = \sqrt{1 + 4 - 3 - 12 + 14} = 2$

hence $r = 2$

$$2h + k - 2r = 0$$