

Question



A particle moves in the x-y plane under the influence of a force such that its linear momentum is $\vec{p}(t) = A(\hat{i} \cos(kt) - \hat{j} \sin(kt))$ where A and k are constants. The angle between the force and momentum is

A 0°

B 30°

C 45°

D 90°

Solution

Correct option is D)

$$P(t) = A(\hat{i} \cos kt - \hat{j} \sin kt)$$

$$F = \frac{dP(t)}{dt}$$

$$F = \frac{d}{dt} A(-\hat{i} \sin kt - \hat{j} \cos kt)$$

$$= A(-\hat{i} \sin kt(k) - \hat{j} \cos kt(k))$$

$$F = Ak(-\hat{i} \sin kt - \hat{j} \cos kt)$$

Angle between two vectors is θ

$$\cos \theta = \frac{F \cdot P}{|F| |P|}$$

$$= A \frac{(\hat{i} \cos kt - \hat{j} \sin kt) \cdot Ak(-\hat{i} \sin kt - \hat{j} \cos kt)}{|F| |P|}$$

$$\frac{A^2 k(-\sin kt \cos kt + \sin kt \cos kt)}{|F| |P|}$$

$$= A^2 k(0) = 0$$

$$\theta = \cos^{-1}(0) = 90^\circ$$