Question 16:

Using integration, find the area of the region bounded by the following curves, after making a rough sketch: $y = 1 + |x + 1|, x = -2, x = 3, y = 0$.

Solution:

To find area enclosed by

$x = -2$, $x = 3$, $y = 0$ and $y = 1 + |x + 1|$

$\Rightarrow \quad y = 1 + x + 1$, if $x + 1 \ge 0$

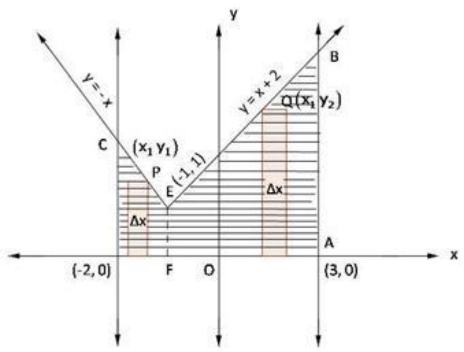
$\Rightarrow \quad y = 2 + x \qquad\qquad ---(1)$, if $x \ge -1$

And $\quad y = 1 - (x + 1)$, if $x + 1 < 0$

$\Rightarrow \quad y = 1 - x - 1$, if $x < -1$

$\Rightarrow \quad y = -x \qquad\qquad ---(2)$, if $x < -1$

So, equation (1) is a straight line that passes through $(0,2)$ and $(-1,1)$. Equation (2) is a line passing through $(-1,1)$ and $(-2,2)$ and it is enclosed by line $x = 2$ and $x = 3$ which are lines parallel to y-axis and pass through $(2,0)$ and $(3,0)$ respectively $y = 0$ is x-axis. So, a rough sketch of the curves is given as:-



Shaded region represents the required area.

So, required area = Region $(ABECDFA)$

Required area = $(\text{region } ABEFA + \text{region } ECDFE) \qquad ---(1)$

region $ECDFE$ is sliced into approximation rectangle with width Δx and length y_1.

Area of those approximation rectangle is $y_1 \Delta x$ and these slids from $x = -2$ to $x = -1$.

Region $ABEFA$ is sliced into approximation rectangle with width Δx and length y_2.

Area of those rectangle is $y_2 \Delta x$ which slides from $x = -1$ to $x = 3$. So, using equation (1),

$$\text{Required area} = \int_{-2}^{-1} y_1 dx + \int_{-1}^{3} y_2 dx$$

$$= \int_{-2}^{-1} (-x)\, dx + \int_{-1}^{3} (x + 2) dx$$

$$= -\left[\frac{x^2}{2}\right]_{-2}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$$

$$= -\left[\frac{1}{2} - \frac{4}{2}\right] + \left[\left(\frac{9}{2} + 6\right) - \left(\frac{1}{2} - 2\right)\right]$$

$$= \frac{3}{2} + \left(\frac{21}{2} + \frac{3}{2}\right)$$

$$= \frac{27}{2}$$

Required area = $\frac{27}{2}$ sq.units