Question 14:

Using integration, find the area of the region bounded by the line $2y = 5x + 7$, x-axis and the lines $x = 2$ and $x = 8$.

Solution:

To find area bounded by x-axis and

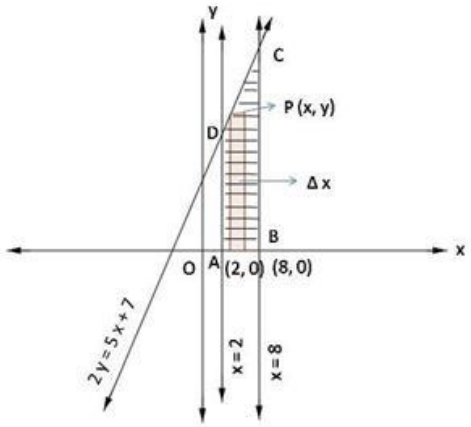
$2y = 5x + 7$ $\quad ---(1)$

$x = 2$ $\quad ---(2)$

$x = 8$ $\quad ---(3)$

Equation (1) represents line passing through $\left(-\frac{7}{5}, 0\right)$ and $\left(0, \frac{7}{2}\right)$ equation $(2), (3)$ shows line parallel to y-axis passing through $(2,0), (8,0)$ respectively.

A rough sketch of curves is as below:-



Shaded region represents the required area. We slice the region into approximation rectangles of Width $=\Delta x$ and length $= y$

Area of the rectangle $= y\Delta x$.

This approximation rectangle slides from $x = 2$ to $x = 8$, so

Required area $=$ (Region $ABCDA$)

$$= \int_2^8 \left(\frac{5x+7}{2}\right) dx$$

$$= \frac{1}{2}\left(\frac{5x^2}{2} + 7x\right)_2^8$$

$$= \frac{1}{2}\left[\left(\frac{5(8)^2}{2} + 7(8)\right) - \left(\frac{5(2)^2}{2} + 7(2)\right)\right]$$

$$= \frac{1}{2}\left[(160 + 56) - (10 + 14)\right]$$

$$= \frac{192}{2}$$

Required area $= 96$ square units