Question 8:

Find the area under the curve $y = \sqrt{6x + 4}$ above x-axis from $x = 0$ to $x = 2$. Draw a sketch of the curve also.

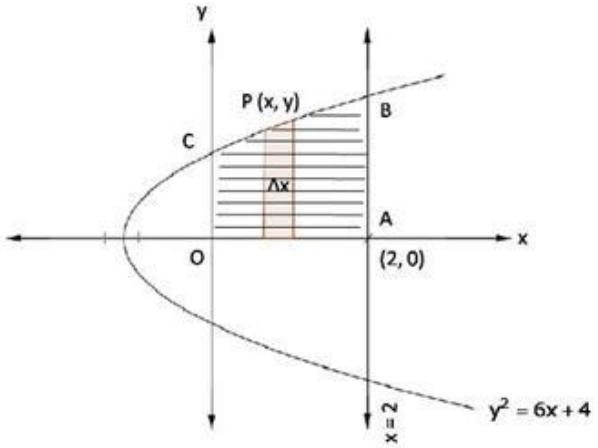
Solution:

We have to find area enclosed by x-axis

$x = 0,\ x = 2$ $\quad ---(1)$

and $y^2 = 6x + 4$ $\quad ---(2)$

Equation (1) represents y-axis and a line parallel to y-axis passing through $(2,0)$ respectively. Equation (2) represents a parabola with vertex at $\left(-\frac{2}{3}, 0\right)$ and passes through the points $(0,2), (0,-2)$, so rough sketch of the curves is as below:-



Shaded region represents the required area. It is sliced in approximation rectangle with its Width $= \Delta x$, and length $= (y - 0) = y$

Area of rectangle $= y\Delta x$.

This approximation rectangle slide from $x = 0$ to $x = 2$, so

Required area = Region $OABCO$

$$= \int_0^2 \sqrt{6x + 4}\,dx$$

$$= \left\{\frac{2}{3}\frac{(6x+4)\sqrt{6x+4}}{6}\right\}_0^2$$

$$= \frac{1}{9}\left[\left((12 + 4)\sqrt{12 + 4}\right) - \left((0 + 4)\sqrt{0 + 4}\right)\right]$$

$$= \frac{1}{9}\left[16\sqrt{16} - 4\sqrt{4}\right]$$

$$= \frac{1}{9}(64 - 8)$$

Required area $= \frac{56}{9}$ square units