Question 7:

Sketch the graph of $y = \sqrt{x + 1}$ in $[0,4]$ and determine the area of the region enclosed by the curve, the x-axis and the lines $x = 0$, $x = 4$.

Solution:
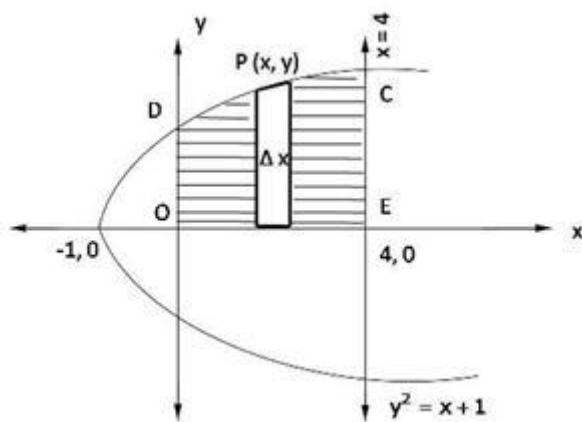
We have to find area enclosed by x-axis and

$$y = \sqrt{x + 1}$$

$$\Rightarrow \quad y^2 = x + 1 \qquad\qquad ---(1)$$

$$\text{and } x = 0 \qquad\qquad ---(2)$$

$$x = 4 \qquad\qquad ---(3)$$

Equation (1) represent a parabola with vertex at $(-1, 0)$ and passing through $(0, 1)$ and $(0, -1)$. Equation (2) is y-axis and equation (3) is a line parallel to y-axis passing through $(4, 0)$. So rough sketch of the curve is as below:-

We slice the required region in approximation rectangle with its Width $= \Delta x$, and length $= y - 0 = y$

Area of rectangle $= y \Delta x$.

Approximation rectangle moves from $x = 0$ to $x = 4$. So

Required area = Shaded region

$$= (\text{Region } OECDO)$$

$$= \int_0^4 y\,dx$$

$$= \int_0^4 \sqrt{x + 1}\,dx$$

$$= \left(\frac{2}{3}(x+1)\sqrt{x+1} \right)_0^4$$

$$= \frac{2}{3}\left[\left((4 + 1)\sqrt{4 + 1} \right) - \left((0 + 1)\sqrt{0 + 1} \right) \right]$$

Required area $= \frac{2}{3}\left[5\sqrt{5} - 1 \right]$ square units