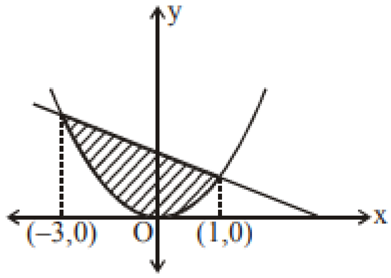
Area under the curve:

Question 1: (JEE Main 2020)

The area (in sq. units) of the region $\{(x,y) \in R^2 : x^2 \le y \le 3 - 2x\}$, is

(1) $\frac{29}{3}$ (2) $\frac{31}{3}$ (3) $\frac{34}{3}$ (4) $\frac{32}{3}$

Sol:



Area $= \int_{-3}^{1} \left(3 - 2x - x^2\right) dx = \frac{32}{3}$

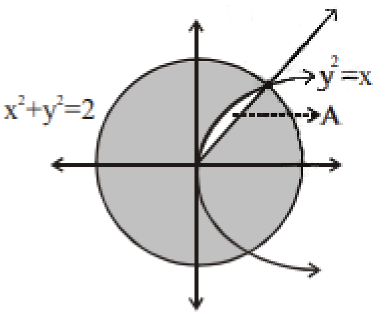
Question 2: (JEE Main 2020)

The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is :

(1) $\frac{1}{3}(12\pi - 1)$ (2) $\frac{1}{6}(12\pi - 1)$

(3) $\frac{1}{6}(24\pi - 1)$ (4) $\frac{1}{3}(6\pi - 1)$

Sol:



$A = \int_0^1 \left(\sqrt{x} - x\right) dx$

$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 = \frac{1}{6}$

Required Area : $\pi r^2 - \frac{1}{6} = \frac{1}{6}(12\pi - 1)$

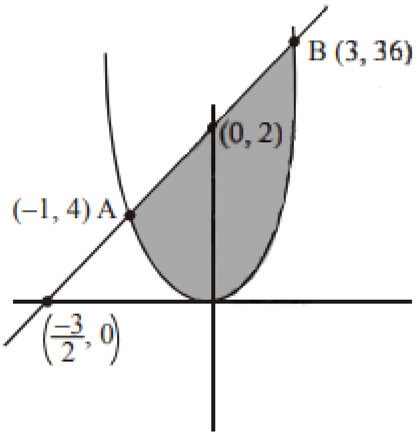
Question 3: (JEE Main 2020)

The area (in sq. units) of the region $\{(x, y) \in R^2 | 4x^2 \le y \le 8x + 12)$ is :

(1) $\frac{127}{3}$ (2) $\frac{125}{3}$ (3) $\frac{124}{3}$ (4) $\frac{128}{3}$

Sol:

$4x^2 - y \le 0$ and $8x - y + 12 \ge 0$



On solving $y = 4x^2$

and $y = 8x + 12$

We get A (–1, 4) & B(3, 36)

Required area = area of the shaded region

$$= \int_{-1}^{3} (8x + 12 - 4x^2)dx = \frac{128}{3}$$

Question 4: (JEE Main 2020)

Given : $f(x) = \begin{cases} x & , \quad 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , \quad x = \frac{1}{2} \\ 1 - x & , \quad \frac{1}{2} < x \le 1 \end{cases}$ and $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in R$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is :

(1) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ (2) $\frac{\sqrt{3}}{4} - \frac{1}{3}$

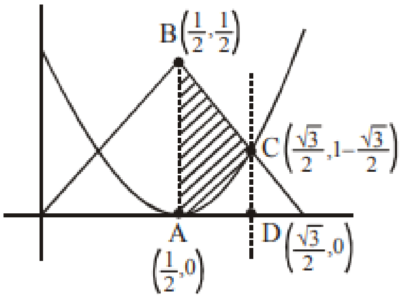
(3) $\frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{1}{2} - \frac{\sqrt{3}}{4}$

Sol:

Required area = Area of trepezium ABCD –

Area of parabola between $x = \frac{1}{2}$ & $x = \frac{\sqrt{3}}{2}$

$$A = \frac{1}{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right) - \int_{1/2}^{\sqrt{3}/2}\left(x-\frac{1}{2}\right)^2 dx = \frac{\sqrt{3}}{4}-\frac{1}{3}$$



Question 5: (JEE Main 2020)

Area (in sq. units) of the region outside $\frac{|x|}{2}+\frac{|y|}{3}=1$ and inside the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is :

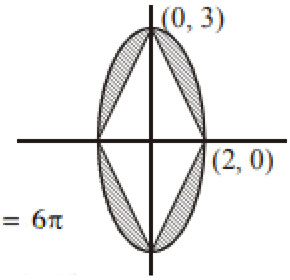
(1) $3(4 - \pi)$ (2) $6(\pi - 2)$

(3) $3(\pi - 2)$ (4) $6(4 - \pi)$

Sol:

$$\frac{|x|}{2}+\frac{|y|}{3}=1$$

$$\frac{x^2}{4}+\frac{y^2}{9}=1$$



Area of Ellipse = $\pi ab = 6\pi$

Required area,

$= \pi \times 2 \times 3$ – (Area of quadrilateral)

$= 6\pi - \frac{1}{2}6\times 4$

$= 6\pi - 12$

$= 6(\pi - 2)$

Question 6:(JEE Main 2020)

Consider a region R = $\{(x, y) \in R^2 : x^2 \le y \le 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true?

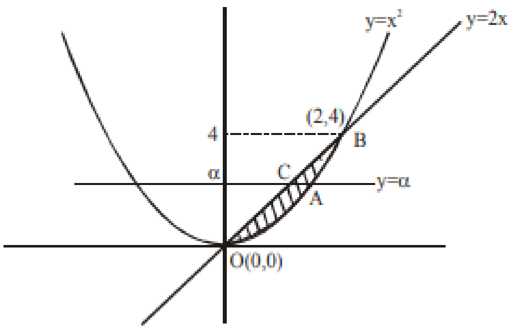
(1) $\alpha^3 - 6\alpha^2 + 16 = 0$

(2) $3\alpha^2 - 8\alpha + 8 = 0$

(3) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$

(4) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$

Sol:



According to ques, area of OABC = 2 area of OAC

$$\Rightarrow \int_0^4 \left(\sqrt{y} - \frac{y}{2}\right) dy = 2\int_0^{\alpha} \left(\sqrt{y} - \frac{y}{2}\right).dy$$

$$\Rightarrow \frac{4}{3} = 2\left[\frac{2}{3}\alpha^{3/2} - \frac{1}{4}.\alpha^2\right]$$

$$\Rightarrow \boxed{3\alpha^2 - 8\alpha^{3/2} + 8 = 0}$$

* $y \ge x^2 \Rightarrow$ upper region of $y = x^2$

$y \le 2x \Rightarrow$ lower region of $y = 2x$

Question 7: (JEE Main 2020)

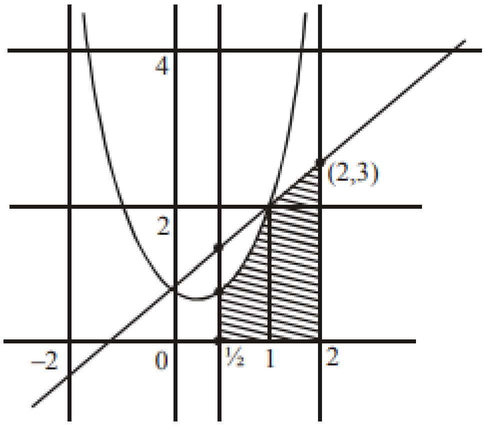
The area (in sq. units) of the region

$\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2\}$ is :

(1) $\frac{79}{16}$ (2) $\frac{23}{6}$ (3) $\frac{79}{24}$ (4) $\frac{23}{16}$

Sol:

$0 \le y \le x^2 + 1,\ 0 \le y \le x + 1,\ \frac{1}{2} \le x \le 2$



$$\text{Required area} = \int_{1/2}^{1} (x^2 + 1)\,dx + \frac{1}{2}(2+3)\times 1$$

$$= \frac{19}{24} + \frac{5}{2} = \frac{79}{24}$$

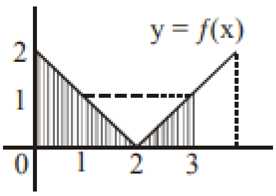
Question 8: (JEE Main 2020)

Let $f(x) = |x - 2|$ and $g(x) = f(f(x)),\ x \in [0, 4]$.

Then $\int_0^3 (g(x) - f(x))\,dx$ is equal to :

(1) $\frac{3}{2}$ (2) 0 (3) $\frac{1}{2}$ (4) 1

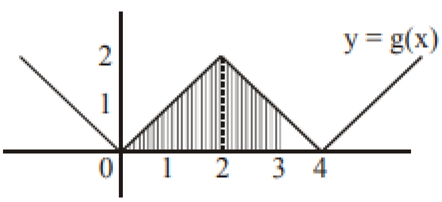
Sol:



$$\int_0^3 g(x) - f(x) = \int_0^3 \big||x-2|-2\big|\,dx - \int_0^3 |x-2|\,dx$$

$$= \left(\frac{1}{2}\times 2\times 2 + 1 + \frac{1}{2}\times 1\times 1\right) - \left(\frac{1}{2}\times 2\times 2 + \frac{1}{2}\times 1\times 1\right)$$

$$= \left(2 + 1 + \frac{1}{2}\right) - \left(2 + \frac{1}{2}\right) = 1$$



Question 9: (JEE Main 2020)

The area (in sq. units) of the region $A = \{(x, y) : (x - 1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\}$, where [t] denotes the greatest integer function, is :

(1) $\frac{8}{3}\sqrt{2}-\frac{1}{2}$ (2) $\frac{8}{3}\sqrt{2}-1$

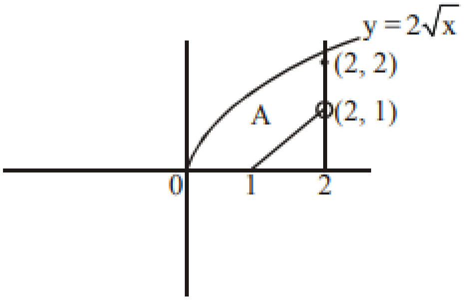
(3) $\frac{4}{3}\sqrt{2}-\frac{1}{2}$ (4) $\frac{4}{3}\sqrt{2}+1$

Sol:

$(x - 1)[x] \le y \le 2\sqrt{x}$, $\boxed{0 \le x \le 2}$

Draw $y = 2\sqrt{x} \Rightarrow y^2 = 4x$ $\boxed{x \ge 0}$

$$y = (x-1)[x] = \begin{cases} 0 & , 0 \le x < 1 \\ x-1 & , 1 \le x < 2 \\ 2 & , x = 2 \end{cases}$$



$$A = \int_0^2 2\sqrt{x}\,dx - \frac{1}{2}1\cdot 1$$

$$A = 2\cdot\left[\frac{x^{3/2}}{(3/2)}\right]_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2}$$

Question 10: (JEE Main 2020)

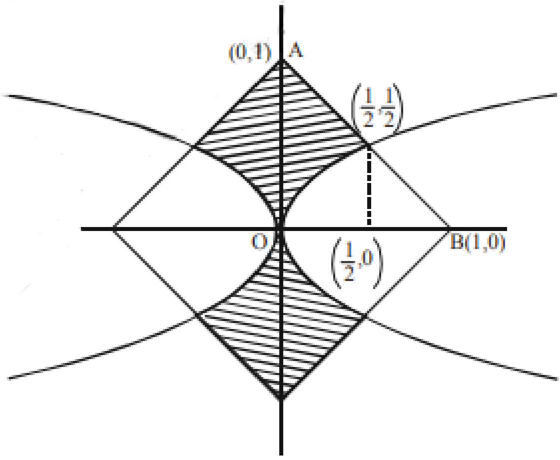
The area (in sq. units) of the region $A = \{(x,y) : |x| + |y| \le 1, 2y^2 \ge |x|\}$ is :

(1) $\frac{1}{6}$ (2) $\frac{1}{3}$ (3) $\frac{7}{6}$ (4) $\frac{5}{6}$

Sol:

$|x| + |y| \leq 1$

$2y^2 \geq |x|$



For point of intersection

$x + y = 1 \Rightarrow x = 1 - y$

$$y^2 = \frac{x}{2} \Rightarrow 2y^2 = x$$

$2y^2 = 1 - y \Rightarrow 2y^2 + y - 1 = 0$

$(2y - 1)(y + 1) = 0$

$$y = \frac{1}{2} \text{ or } -1$$

Now Area of $\Delta OAB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Area of Region $R_1 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Area of Region $R_2 = \frac{1}{\sqrt{2}} \int_0^{\frac{1}{2}} \sqrt{x}\, dx = \frac{1}{6}$

Now area of shaded region in first quadrant

= Area of $\Delta OAB - R_1 - R_2$

$$= \frac{1}{2} - \left(\frac{1}{6}\right) - \left(\frac{1}{8}\right) = \frac{5}{24}$$

So required area $= 4\left(\frac{5}{24}\right) = \frac{5}{6}$