Area under the curve:

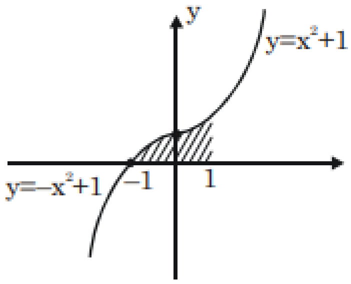
Question 1:(JEE Main 2019)

The area of the region $A=\left[(x,y):0\le y\le x|x|+1 \text{ and } -1\le x\le 1\right]$ in sq. units, is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) 2 (4) $\frac{4}{3}$

Sol:

The graph is a follows



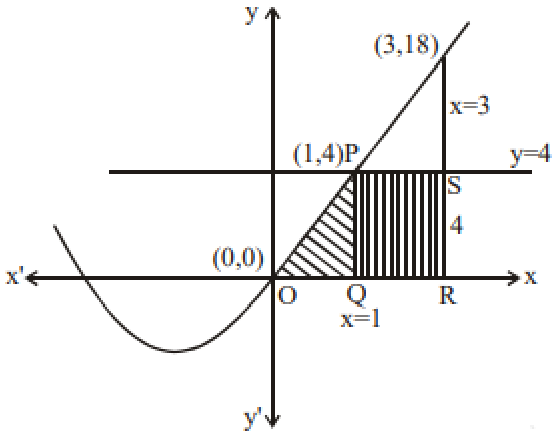
$$\int_{-1}^{0}\left(-x^2+1\right)dx+\int_{0}^{1}\left(x^2+1\right)dx=2$$

Question 2: (JEE Main 2019)

The area (in sq. units) of the region $A = \{(x, y) \in R \times R | 0 \le x \le 3,\ 0 \le y \le 4,\ y \le x^2 + 3x\}$ is :

(1) $\frac{53}{6}$ (2) $\frac{59}{6}$

(3) 8 (4) $\frac{26}{3}$



Sol:

Required Area

$$=\int_{0}^{1}(x^2+3x)dx+\text{Area of rectangle PQRS}$$

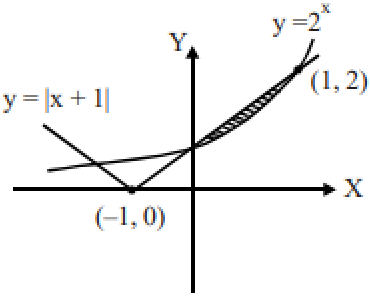
$$=\frac{11}{6}+8=\frac{59}{6}$$

Question 3: (JEE Main 2019)

The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

(1) $\frac{3}{2} - \frac{1}{\log_e 2}$ (2) $\frac{1}{2}$

(3) $\log_e 2 + \frac{3}{2}$ (4) $\frac{3}{2}$



Sol:

Required Area

$$\int_0^1 \left((x+1) - 2^x\right) dx$$

$$= \left(\frac{x^2}{2} + x - \frac{2^x}{\ln 2}\right)_0^1$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{\ln 2}\right) - \left(0 + 0 - \frac{1}{\ln 2}\right)$$

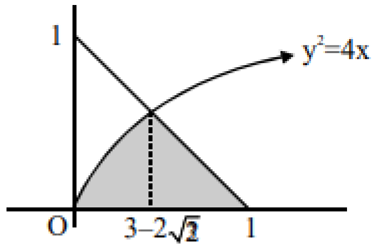
$$= \frac{3}{2} - \frac{1}{\ln 2}$$

Question 4: (JEE Main 2019)

If the area (in sq. units) of the region $\{(x, y) : y^2 \le 4x,\ x + y \le 1,\ x \ge 0,\ y \ge 0\}$ is $a\sqrt{2} + b$, then $a - b$ is equal to :

(1) $\frac{8}{3}$ (2) $\frac{10}{3}$ (3) 6 (4) $-\frac{2}{3}$

$\{(x, y) : y^2 \le 4x,\ x + y \le 1,\ x \ge 0,\ y \ge 0\}$



Sol:

$$A \int_0^{3-2\sqrt{2}} 2\sqrt{x}\,dx + \frac{1}{2}\left(1-\left(3-2\sqrt{2}\right)\right)\left(1-\left(3-2\sqrt{2}\right)\right)$$

$$= \frac{2[x^{3/2}]_0^{3-2\sqrt{2}}}{3/2} + \frac{1}{2}\left(2\sqrt{2}-2\right)\left(2\sqrt{2}-2\right)$$

$$= \frac{8\sqrt{2}}{3} + \left(-\frac{10}{3}\right)$$

$a = \frac{8}{3}$, $b = -\frac{10}{3}$

$a - b = 6$

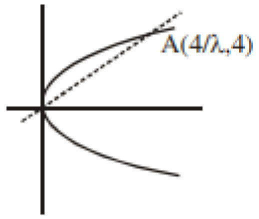
Question 5: (JEE Main 2019)

If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to :

(1) 24 (2) 48

(3) $4\sqrt{3}$ (4) $2\sqrt{6}$

Sol:



$$\text{Area } = \frac{1}{9} = \int_0^{\frac{4}{\lambda}} \left(\sqrt{4\lambda x} - \lambda x\right) dx$$

$$\Rightarrow \lambda = 24$$

Question 6: (JEE Main 2019)

Let $S(\alpha) = \{(x,y) : y^2 \leq x, 0 \leq x \leq \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals

(1) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (2) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

(3) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (4) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$

Sol:

$$S(\alpha) = \{(x,y) : y^2 \leq x, 0 < x \leq \alpha\}$$

$$A(\alpha) = 2\int_0^{\alpha} \sqrt{x}\,dx = 2\alpha^{3/2}$$

$$A(4) = 2 \times 4^{3/2} = 16$$

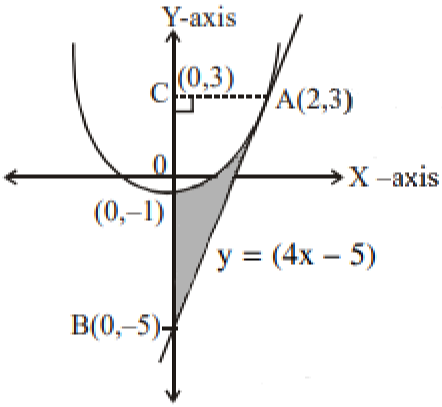
$$A(\lambda) = 2 \times \lambda^{3/2}$$

$$\frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \lambda = 4.\left(\frac{4}{25}\right)^{1/3}$$

Question 7: (JEE Main 2019)

The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is :

(1) $\frac{14}{3}$ (2) $\frac{56}{3}$ (3) $\frac{8}{3}$ (4) $\frac{32}{3}$



Sol:

Equation of tangent at (2,3) on $y = x^2 - 1$, is $y = (4x - 5)$ ....(i)

$\therefore$ Required shaded area

$$= \text{ar}(\Delta ABC) - \int_{-1}^{3} \sqrt{y+1}\, dy$$

$$= \frac{1}{2}.(8).(2) - \frac{2}{3}\left((y+1)^{3/2}\right)_{-1}^{3}$$

$$= 8 - \frac{16}{3} = \frac{8}{3} \text{ (square units)}$$